

A fully abstract semantics for concurrent graph reduction

$A \approx_{\text{EFF}} B \iff EY$

$AB \xrightarrow{\lambda} AC \iff$ s p p r p r s n t s u str t s nt s or v r nt t unt p λ u us
t r urs v r tonsⁿ r st pr s nt su r o st n or on u str ton ort un
t p λ u us on ntr t n on AB A Y n G s or ont λ u us AB A Y
n G s or s s on t ost out r ostr u ton t outs r n s s not n nt
n n p nt tons o s r n r u n s nt

1 Introduction

sp pr s outt r tons p t nt oⁿ s o o put rs n full
abstraction, n concurrent graph reduction Fu str ton st stu or t
n not ton n opr r ton s nt s Con urr nt p r u ton s n
nt p r p nt t on t n qu or non str t un ton pro n
n s
nt sp pr pp t t n qu s AB A Y n G
to pr s nt u str t not ton s nt sort on urr nt p r u
ton or t n n EY E st t oo
n on so us to s ro u str ton, o p r p nt ton,
n on urr n t or -

1.1 Full abstraction

Fu str ton, or nⁿ E, p or st r tons p

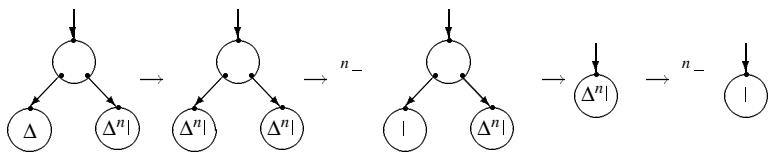
t s v op AD H s n p nt ton ost
 out r ostr u ton H o s r t t ost out r ostr u ton nt
 pon nt t to u t n pr ss on, u to oss sharing nor ton For
 p , t n

$$I = \lambda x. x \quad \Delta = \lambda x. xx \quad M \cdot N = N \quad M^{n+} N = M(M^n N)$$

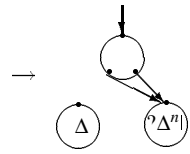
nt v u ton $\Delta^{n+} I \rightarrow^* I$ s

$$\Delta^{n+} I \rightarrow (\Delta^n I)(\Delta^n I) \rightarrow^{n-} I(\Delta^n I) \rightarrow \Delta^n I \rightarrow^{n-} I$$

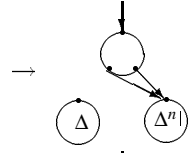
us $\Delta^n I$ t s $n -$ r u tons to tr nt s pon nt o up s
 us op n $\Delta^n I$ nt r u ton $\Delta^{n+} I \rightarrow (\Delta^n I)(\Delta^n I)$, n n r
 s n r t s nt tr sort sr u ton, r not sun ton
 pp t on



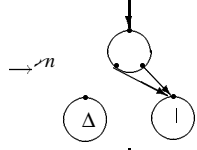
s n n s us t p nt ton β r u ton t su st tu
 ton n r u $(\lambda w. M)N \rightarrow M[N/w]$, spr t op N or
 o ur n w n M , n op t n sto r u spr t
 nr o v t s n n r t r t n op n tr s op pointers to
 tr s t t s r u s nt graphs r t r t ns nt trees For p , t



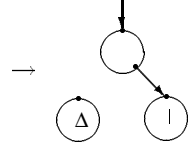
up t n



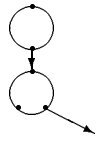
p n tr v rs



n u t on



not *con uent* or *Church Rosser*, s n sp n tr v rs



- Gr o t on s s nt un port nt, so p n on v r t n on v r t out r o t n n ou p tt sto tru , sn r o ton s ntro u on us or t tons
 - n s s nt un port nt, so p n on v r rr sp t v t r ts no s r t or not n p r t u r t s nst t on urr nt v u ton s s nt qu v nt to s qu nt v u ton
 - r nt tr nsp r n , nst t t s s nt un port nt p ont ns op no , or po nt r to no r r nu r o pp tons or u str ts nt s
- EY GC ZA - Anu r o o p rs non str t un ton n s, not H s o p r ort G n, us opt tons n opt rs, not *peephole optimizers* EY E, C r p on s tr t not rs nt qu v nt, ut or nt tr s nt s s orr t t n no t t n su opt ton v t s op r ton v our n ont ts
- ortun t , t s nt s s not o p t, t nt r v op t tonst tr nots nt qu v nt, n t r s t pt ton ort o p r r r to us *ad hoc* son n ato ust s nt n v opt ton on t roun st tt s nt s s too n t s nt s s u

2 Tree reduction

s C p t r p r s n t s s u r c e s t n o r o n u s t r t o s
 or t o s t o u t r o s t r u t o n t u n t p λ u u s t o n n t r t s o n
 A B A Y n G s or o n t λ u u s u t s o
 n u s t r o A B A Y B A E D E G B A A
 D E G et al B D E C E n

2.1 The λ-calculus with P

n t s C p t r s u s s t t o r v o p A B A Y n G
 s o n l e f t m o s t o u t e r m o s t r u t o n s s t s n t s s o t n o n
 s t r i c t u n t o n n u s s u s A G s F A B s
 o n r E s G o r E s r n n H s
 H D A et al
 n t u n t p λ u u s p r s s o n s r u n t o n s n t s u n t o n s
 t u n t o n s s n p u t s n r t u r n o t r u n t o n s n r r t s s p u r
 t o r o p u t t o n s t r t o n s r t o n s t
 u n t p λ u u s s t r o r s o p r s s o n

- A free variable x
- An application MN
- An abstraction $\lambda x.M$

u t r s r s e q u e n t i a l n t o n o r o p u t t o n s β r u t o n r
 n s t r t o n s p p (λx.M)N → M[N/x] F o o n s
 o u p t t t n n u s t r s n t s u s p r
 s o o r o p r o p u t t o n r r n u r o p o s s p r
 o n t o r s o n n u s p r o n t o n A B A Y
 n G u s p r o n v r n n B D

ours \forall s to ω continuous unctns, t t s

$$a \text{ st } \quad t \circ a_{\omega} \leq a \leq \dots$$

t n

$$fa \text{ st } \quad t \circ fa_{\omega} \leq fa \leq \dots$$

For \forall p, t s rst odd unctns n

$$- \text{ st } \quad t \circ \omega \leq - \leq \omega \leq \dots$$

ut

$$\omega \text{ s not t } \quad t \circ - \leq \omega \leq \omega \leq \dots$$

\prod not ton s nt s $\Lambda_p \text{ n } \mathbf{D} \simeq (\mathbf{D} \rightarrow \mathbf{D})_{\perp}$ os o t t su
 \mathbf{D} ust \forall st, pr s nt t st t s qu n ω \prod t o ns $\mathbf{D}_{\omega}, \mathbf{D}, \dots$
r

$$\mathbf{D}_{\omega} = \mathbf{D}_{n+} = (\mathbf{D}_n \rightarrow \mathbf{D}_n)_{\perp}$$

s n so pr s nt st \forall point ω functor F t n o ns

$$F\mathbf{D}_i = (\mathbf{D}_i \rightarrow \mathbf{D}_i)_{\perp} = \mathbf{D}_{i+}$$

n n or r to s o t t \mathbf{D} \forall st, s o t t F s ont nuous n or r to o
t s, pr s nt

- A not on ω domain, su t tt on po nt o n s o n, n F s un tor t n o ns
- A not on ω order t n o ns t st nt n r \forall r n o ns s t
- A not on ω continuous functor t n o ns, su t t F s ont nuous

Fo o n ω , us t category of ω cpo s with embeddings
st pprop r t not on ω or r o ns n F s ont nuous un tor, t

ust \forall st \forall point, us sour \prod t on ω \mathbf{D}
r st t s s ton pr s nt t t n t s o t s onstru tion
s n t s or r n r so s p t or t or nt r st

EXA. $E \dashv \text{lift } C \rightarrow C_{\perp}$ s functors n v

- no $\text{t lift } A \text{ in } C_{\perp}$ or $A A$

∇ e^R s un qu \mathbb{P}_n , so $e A \rightarrow B$ in ωCPOE n $f B \rightarrow A$ in ωCPOE
 t n

$$(e \circ f \leq \text{id}, f \circ e = \text{id}) \quad p \quad s \quad e^R = f$$

$(\perp)_{\omega\text{CPOE}} \rightarrow \omega\text{CPOE}$ st ω n ω un tor t

- A_{\perp} in ωCPOE or A in ωCPOE^-
- $e_{\perp} A_{\perp} \rightarrow B_{\perp}$ in ωCPOE or $e A \rightarrow B$ in ωCPOE^-

$\Delta \omega\text{CPOE} \rightarrow \omega\text{CPOE}$ st ω n ω un tor t

- $\Delta A = (A, A)$ in ωCPOE or A in ωCPOE^-
- $\Delta f = (f, f) \Delta A \rightarrow \Delta B$ in ωCPOE or $f A \rightarrow B$ in ωCPOE^-

$(\rightarrow)_{\omega\text{CPOE}} \rightarrow \omega\text{CPOE}$ st ω ont nuous ω un t on sp ω un tor t

- $(A \rightarrow B)$ in ωCPOE or (A, B) in ωCPOE^-
 - $(e \rightarrow f) (A \rightarrow B) \rightarrow (A' \rightarrow B')$ in ωCPOE or $(e, f) (A, B) \rightarrow (A', B')$ in ωCPOE^-
- $r \quad e \rightarrow f \quad s \quad \mathbb{P}_n$

$$(e \rightarrow f)g = f \circ g \circ e^R$$

$$(e \rightarrow f)^R g = e \circ g \circ f^R$$

st n t o t n ωCPOE^- □

DEF ∇ A o on $\{e_i A_i \rightarrow A \mid i \text{ in } \omega\}$ s *determined* ω
 $\bigvee \{e_i \circ e_i^R \mid i \text{ in } \omega\} = \text{id}$ □

ω ∇ \mathbb{P}_n Any determined cocone is a colimit.

ω F^- t $\{e_i A_i \rightarrow A \mid i \text{ in } \omega\}$ t r n o on ω n ω n
 $\{e_i^j A_i \rightarrow A_j \mid i \leq j \text{ in } \omega\}$ n or n ot r o on $\{f_i A_i \rightarrow B \mid i \text{ in } \omega\}$, \mathbb{P}_n
 $g A \rightarrow B$ s

$$g = \bigvee \{f_i \circ e_i^R \mid i \text{ in } \omega\}$$

$$g^R = \bigvee \{e_i \circ f_i^R \mid i \text{ in } \omega\}$$

∇ n n s o t t g s t un qu n ω s u t t g o e_i = f_i ∇ us
 $\{e_i A_i \rightarrow A \mid i \text{ in } \omega\}$ s o t □

ω Any ω chain in ωCPOE has a determined cocone.

ω F^- t $\{e_i^j A_i \rightarrow A_j \mid i \leq j\}$ n ω n An instantiation ω t s n
 s un t on f s u t t

$$\text{dom } f = \omega \quad f_i \in A_i \quad e_i^{jR}(f_j) = f_i$$

t n \mathbb{P}_n

$$A = \{f \mid f \text{ s n nst nt t on}\}$$

t t p o n t s o r r n ∇ s s n ω p o t o n

$$\bigvee \{f_i \mid i \text{ in } \omega\} j = \bigvee \{f_i j \mid i \text{ in } \omega\}$$

∇ n \mathbb{P}_n

$$e_i a j = \begin{cases} e_i^j a & i \leq j \\ e_i^{jR} a & \text{ot r s} \end{cases}$$

$$e_i^R f = f i$$

n s o t t $\{e_i A_i \rightarrow A \mid i \text{ in } \omega\}$ s t r n o on ω □

DEF ∇ \mathbf{D} st t r n o t ω t ω n

$$\mathbf{D}_{\perp} =$$

$$\mathbf{D}_{i+} = (\mathbf{D}_i \rightarrow \mathbf{D}_i)_{\perp}$$

t $e_i \mathbf{D}_i \rightarrow \mathbf{D}$ in ωCPOE ω n r o p o s t o n ∇ n \mathbf{D} s t n t ω \mathbb{P}_n
 p o n t ω t ω un tor $(\perp)_{\omega} \circ (\rightarrow)_{\omega} \Delta$ ω n r o p o s t o n ω □

2.6 Logical presentation of D

n t o n ω , ω ω n s t r t p r s n t t o n ω \mathbf{D} , u s n ω t t ω or ω ω
 p o s t n ω n t s s t o n, p r o ω n r t p r s n t t o n ω \mathbf{D} ,
 s r t o c ω s *information systems* Fo o n ω \mathbf{A} , \mathbf{A} , Y s
domain theory in logical form u s t p r o ω o ω Φ s n t r n t ω p r
 s n t t o n ω \mathbf{D}^- n p r t u r, s o t t t ω p o ω l t e r s ω Φ s q u ω n t
 to \mathbf{D}^-

DEF ∇ $\Psi \subseteq \Phi$ s l t e r ω

- $\omega \in \Psi$
- $\omega \in \Psi$ n $\vdash \phi \leq \psi$ t n $\psi \in \Psi$
- $\omega \in \Phi, \Psi$

- $\vdash \phi \leq \psi \iff [[\phi]] \leq [[\psi]]$
- $a \leq \omega$

- For all a in \mathcal{D} , $a \leq \perp$ -

$\perp = \bigvee \emptyset$

$$a = \perp = \bigvee \emptyset = \bigvee \{b \mapsto c \mid b \mapsto c \leq a\}$$

Therefore, \perp is the least element

$$\text{apply } a d = \text{apply}(\bigvee \{b \mapsto c \mid b \mapsto c \leq a\})d$$

so $a = \bigvee \{b \mapsto c \mid b \mapsto c \leq a\}$

- $a \mapsto b \leq \bigvee C$ for nonempty $C \subseteq \mathcal{D}$ -

$$b = \text{apply}(a \mapsto b)a \leq \text{apply}(\bigvee C)a = \bigvee \{\text{apply } ca \mid c \in C\}$$

Therefore, $c \in C$ implies $b \leq \text{apply } ca$ so

$$a \mapsto b \leq a \mapsto \text{apply } ca \leq c$$

Thus $a \mapsto b$ is the least

- $a \mapsto b \leq \bigvee A$ for \mathcal{D} -

$$b = \text{apply}(a \mapsto b)a \leq \text{apply}(\bigvee A)a = \bigvee \{\text{apply } ca \mid c \in A\}$$

Therefore, $c \in A$ implies $b \leq \text{apply } ca$

$$\begin{array}{l} \Rightarrow \forall x. \Gamma \vdash \lambda x. M \quad \psi_i \rightarrow \chi_i \quad \rightarrow I \\ \Rightarrow \Gamma \vdash \lambda x. M \quad \psi_i \rightarrow \chi_i \quad \leq \end{array}$$

us $(\wedge I)$ n $(\leq), \Gamma \vdash \lambda x. M \quad \phi^- \quad \square$

is a standard result in proof theory or presentations of
 lambda calculus, so that the notion of substitution is
 not too difficult to understand in the theory

$(M \sqsubseteq_D N \Rightarrow M \sqsubseteq_S N)$ For $n \Gamma \vdash \phi, \sigma \in M \sqsubseteq_D N$ t n

$$\begin{aligned} & \Gamma \vdash M \phi \\ & \Rightarrow \llbracket \phi \rrbracket \leq \llbracket M \rrbracket \llbracket \Gamma \rrbracket \\ & \Rightarrow \llbracket \phi \rrbracket \leq \llbracket N \rrbracket \llbracket \Gamma \rrbracket \\ & \Rightarrow \Gamma \vdash M \phi \end{aligned}$$

ropt
H pot s s
ropt

us $\sigma \in M \sqsubseteq_D N$ t n $M \sqsubseteq_S N$

$(M \sqsubseteq_S N \Rightarrow M \sqsubseteq_D N)$ For $n \sigma, \sigma \in M \sqsubseteq_S N$ t n

$$\begin{aligned} & \llbracket M \rrbracket \sigma \\ & = \bigvee \{ \llbracket \phi \rrbracket \mid \Gamma \vdash M \phi \} \end{aligned}$$

- $\text{rec}D$ in M s *recursive declaration* $\hookrightarrow D$ n M^-

EXA. E -

- $x = M$,

pp t on M to ts \leftarrow , t s r n \leftarrow n r n

$$x = u \cdot v,$$

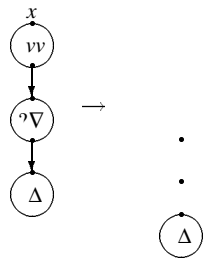
$$u = \nabla z,$$

$$v = \nabla z,$$

$$z = \nabla M$$



⊗ p



DEF \dashv \dashv s \dashv n \dashv o s

- (BUILD) $x = (\text{rec } D \text{ in } M) \mapsto \text{local } D \text{ in } (x = M)$
- (∇ TRAV) $x = \nabla y, y = ?M \mapsto x = \nabla y, y = M$
- (TRAV) $x = y \ z, y = ?M \mapsto x = y \ z, y = M$
- (\forall TRAV) $x = y \forall z, y = ?M \mapsto x = y \forall z, y = M$
- (∇ UPD) $x = \nabla y, y = \lambda w. M \mapsto x = \lambda w. M, y = \lambda w. M$
- (UPD) $x = y \ z, y = \lambda w. M \mapsto x = M[z/w], y = \lambda w. M$
- (\forall UPD) $x = y \forall z, y = \lambda w. M \mapsto x = \lambda w. M, y = \lambda w. M$
- (γ) $v(\text{wv } D) . D \mapsto \varepsilon$

n stru tur ru s

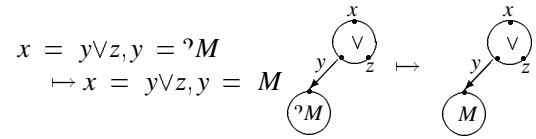
$$(L) \frac{D \mapsto E}{D, F \mapsto E, F} \quad (R) \frac{D \mapsto E}{F, D \mapsto F, E} \quad (v) \frac{D \mapsto E}{\text{vx} . D \mapsto \text{vx} . E}$$

ot t $D \mapsto E$ t n rv $D \supseteq rv E$ n wv $D = wv E$

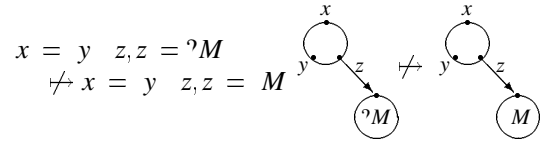
- $D \mapsto E \dashv D \equiv \mapsto \equiv E$
- $D \mapsto \sim E \dashv D \equiv E$, n $D \mapsto^{n+} E \dashv D \mapsto \rightarrow^n E$
- $D \mapsto^* E \dashv \exists n . D \mapsto^n E$
- $D \mapsto^{\leq i} E \dashv \exists n \leq i . D \mapsto^n E$

□

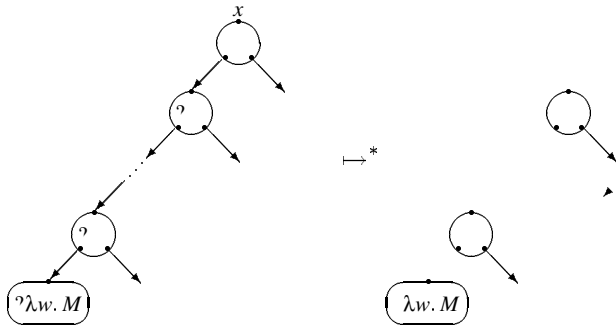
EXA E



ot t t s n r o n v u t o n, v

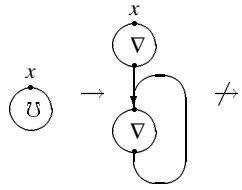


s p s s s p n t r v r s u s t r e t n s p n e u n
 t n r t o n, p p t o n, n o r n o s, t For p



Ho v r t o or r o t on n v o v s p s r tr r s ,
n so s u r r r nu r t , n so ss s op or on urr n - n p

n r n p s



$$\text{set } Xfg\sigma x = \begin{cases} f(g\sigma)x & \text{if } x \in X \\ \sigma x & \text{otherwise} \end{cases}$$

$$\text{fix } f = \bigvee \{f^n \perp \mid n \text{ in } \omega\}$$

- $M \sqsubseteq_D N \iff \llbracket M \rrbracket \leq \llbracket N \rrbracket^-$
- $D \sqsubseteq_D E \iff \text{wv } D = \text{wv } E \text{ and } \llbracket D \rrbracket \leq \llbracket E \rrbracket^-$ □

EXA. E^- is not the set of terms \perp, sn

$$\llbracket \text{rec } x = \lambda y. \nabla x \text{ in } \nabla x \rrbracket$$

$$= \llbracket \nabla x \rrbracket$$

↪ y s t s $\overset{\Pi}{\neq}$ s Ψ t n

$$\begin{aligned} w &= \lambda y. w, x = \lambda y. w, z = x \ y, D \\ &\rightarrow w = \lambda y. w, x = \lambda y. w, z = \nabla w, D \\ &\rightarrow w = \lambda y. w, x = \lambda y. w, z = \lambda y. w, D \end{aligned}$$

n u t o n s t s $\overset{\Pi}{\neq}$ s z χ us

$$(w = \lambda y. w, x = \lambda y. w) \ \phi$$

Fro t s t s s p t o s o t t (w = \lambda y. w) (w \ \phi)^-

↙ s $\overset{\Pi}{\neq}$ t o n p n s o n t n o t o n \ominus p ∇ t n s o n, s t p r o r r $D \sqsubseteq E^-$

DEF ↙ $-D \sqsubseteq E \ominus$ n $\overset{\Pi}{\neq}$ $\bar{x}_i \bar{y}_i D'$ n E' s u t t $D \equiv v\bar{x}. D'$ $E \equiv v\bar{y}. (D', E')$ f v $D \cap \bar{y} = \emptyset$

o t t t \sqsubseteq s p r o r r i n t t $D \sqsubseteq E \sqsubseteq D \ominus D \equiv E^-$ □

n t n $\overset{\Pi}{\neq}$ t t o p r t o n n t r p r t t o n \ominus t o \ominus^-

DEF ↙ -For o s r t o n s, $\models D \ \Delta$ s ∇ n t ∇ o s $(\varepsilon I) \models D \ \varepsilon$ $(\omega I) \models D \ (x \ \omega)$

n s t r u t u r r u s

$$(\wedge I) \frac{\models D \ \Gamma \ \models D \ \Delta}{\models D \ \Gamma \wedge \Delta} \quad (\rightarrow I) \frac{\forall (z = x \ y) \sqsubseteq E \supseteq D. \ D \Downarrow_x \models E \ (y \ \phi) \Rightarrow \models E \ (z \ \psi)}{\models D \ (x \ \phi \rightarrow \psi)}$$

↙ s n ∇ n r ∇ t o n D $\overset{\Pi}{\neq}$ n ∇ $\Gamma \models D \ \Delta \ominus$ $\forall E. (\models D, E \ v(wvD). \Gamma) \ \text{p s} (\models D, E \ \Delta)$

r, $\Gamma \models M \ \phi \ominus$

$$\forall D, z. (\models (D, z = M) \ \Gamma) \ \text{p s} (\models (D, z = M) \ (z \ \phi))$$

n o n s q u n \ominus u s t r t o n s t t o r λ u u s t r s, t s o p r t o n $\overset{\Pi}{\neq}$ t o n ∇ s t t $\overset{\Pi}{\neq}$ t o n \ominus t o n $--$ □

n $\overset{\Pi}{\neq}$ p r o c s s t \ominus o r L a m s \ominus o r Λ_p^- s u s s t s ∇ p r o p o s t o n s, n ∇ u ∇ n t s \ominus t \ominus o r $\Gamma \vdash M \ \phi$ n $\Gamma \vdash D \ \Delta^-$ n \ominus r n t n t p r o c s s t \ominus o r L a m n t t \ominus Λ_p s t p r o c s s t \ominus o r r u r s ∇ r t o n s^- o t t t

• p r o c r u s () n (?) \ominus o r t ∇ n u n t ∇ r t o n s r t s $--$ n t t r s n o \ominus r n t n t ∇ o r n u n t ∇ n o,

on π u t s s $n \phi = \psi \rightarrow \chi$

n n
()

≡

$$\partial[D, E] = (X \cup X', Y \cup Y', Z \cup Z', f \cup f')$$

$$\partial[\forall x. D] = (X \setminus \{x\}, Y \cup \{x\}, Z, f)$$

$$\partial[D] = (X, Y, Z, f), \partial[E] = (X', Y', Z', f') \quad \text{if } X, Y, X' \text{ and } Y' \text{ are disjoint}$$

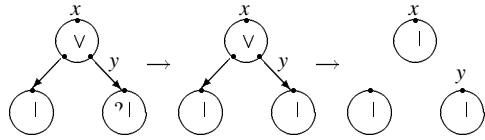
s

□

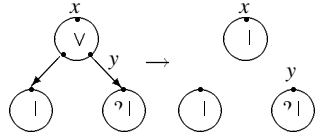
not a theorem

For λ p ,

o t r u t o n



ut not



o t t n t r λ o s t o p t λ o

$$(x = y\forall z, y = \lambda w.M) \mapsto (x = l, y = \lambda w.M)$$

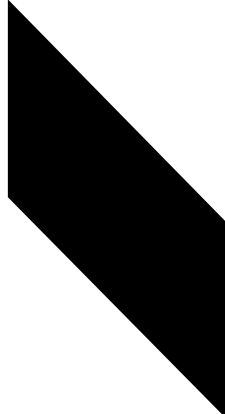
s n \forall to o n s r t s s n $x = z, y = z$ n $x \neq z \neq y$

DEF \mapsto_c s \forall n λ o s

- (BUILD) $x = (\text{rec } D \text{ in } M) \mapsto_c \text{rec } D \text{ in } (x = M)$
- (\forall TRAV) $x = \forall y, y = ?M \mapsto_c x = \forall y, y = M$
- (λ TRAV) $x = y \ z, y = ?M \mapsto_c x = y \ z, y = M$
- (\forall TRAV) $x = y\forall z, y = ?M \mapsto_c x = y\forall z, y = M$
- (\forall UPD) $x = \forall y, y = \lambda w.M \mapsto_c x = \lambda w.M, y = \lambda w.M$
- (λ UPD) $x = y \ z, y = \lambda w.M \mapsto_c x = M[z/w], y = \lambda w.M$
- (\forall UPD a) $x = y\forall z, y = \lambda w.M, z = N \mapsto_c x = l, y = \lambda w.M, z = N$
- (\forall UPD b) $x = y\forall y, y = \lambda w.M \mapsto_c x = l, y = \lambda w.M$
- (\forall UPD c) $x = y\forall x, y = \lambda w.M \mapsto_c x = l, y = \lambda w.M$

n s t r u t u r r u s

$$(L) \frac{D \mapsto_c E}{D, \mathbb{F} \mapsto_c E, F} \quad (R) \quad D$$



- For closed D

- \leq_{γ} is a partial order

- $D \leq_{\gamma} E$ iff $D \equiv_{\gamma} E$

- If $D \rightarrow_{\gamma} E$ then $D \rightarrow_{\gamma} E$

- If $D \leq_{\gamma} E$ then $D \rightarrow_{\gamma} E$

- If $D \leq_{\gamma} E$ then $D \rightarrow_{\gamma} E$

- If $D \rightarrow E$ then $D \rightarrow_{\gamma} E$

- If $D \rightarrow^* E$ then $D \rightarrow_{\gamma}^* E$

- F^-

- \exists n t on \leq_{γ} s r $\{ \}$ \forall \exists n u t on on t p r o c $D \leq_{\gamma} E$, n
s o t t $D \leq_{\gamma} E \leq_{\gamma} D$ t n $D =$

$\rightarrow_c \forall \vec{x} \vec{y}. (F', z = M, G)$	$\forall \text{TRAV}$
$\rightarrow_c \forall \vec{x} \vec{y}. (F', z = M, H)$	$\forall \text{UPDA}$
$\equiv \forall \vec{x}. (\forall \vec{y}. (F', z = M), H)$	VMIG
$\leq_c \forall \vec{x}. (\forall \vec{y}. (F', z = M), H)$	$\text{D} \bullet \text{n} \bullet \leq_c$
$\equiv \forall \vec{x}. (F, H)$	Eqn
$\equiv E$	Eqn

\swarrow
 $\text{us}_c D \rightarrow_c^* \rightarrow_{\gamma}^* \leq_c E^-$
(Others) \swarrow $\text{ot r } \forall \text{o s r } \forall \text{o s } \bullet \rightarrow_c \text{ n so } D \rightarrow_c^* \rightarrow_{\gamma}^* \leq_c E^-$
 $\rightarrow_c D \rightarrow_c^n E_c$ n pro n u t on on n
(

- If $D \equiv (D', x = \lambda w. M) \rightarrow_c E$ then $E' \equiv (E', x = \lambda w. M)$
 - If $D \equiv (D', x = ?M) \rightarrow_c E$ then $E \equiv (D', x = M)$
 or $E \equiv (E', x = ?)$

in position \downarrow - \uparrow r

- $$H \equiv (G, \text{local } K \text{ in } x = M)$$

n s

E

$$\equiv v\vec{x}.(G, J)$$

$$\equiv v\vec{x}.(G, \text{local } K \text{ in } x = M)$$

$$\equiv v\vec{x}.H$$

$$\equiv F$$

Eqn ✓

Eqn

Eqn

Eqn

- or v

$$H \equiv (L, x = \text{rec } K \text{ in } M)$$

n or n N

$$(G, x$$

r u t o n s n n o r r t o v u t x

- If $D \vdash x \prec y$ then $D, E \vdash x \prec y$

- If $\forall x. D \vdash y \prec z$ then $D \vdash y \prec z$

$\dashv\vdash$ If $x \neq y \neq z$ w is fresh and $D \vdash x \prec z$ then $[w/y]D[w/y] \vdash x \prec z$

\Rightarrow Finiteness on $\text{proc } \alpha \prec$ □

not $D \rightarrow_x E$ is a reduction on $\text{spn } \alpha D$

$\dashv\vdash$ $D \rightarrow_x E$ iff $D \equiv \forall \vec{x}. F, E \equiv \forall \vec{x}. G, F \rightarrow_y G$ is an axiom and $F \vdash x \prec y$

\Rightarrow Finiteness

\Rightarrow An axiom on $\text{proc } \alpha D \rightarrow_x E$

\Leftarrow An axiom on $\text{proc } \alpha F \vdash x \prec y$ □

$\dashv\vdash$ If $D \vdash x \prec y$ and $D \rightarrow_y E$ then $D \rightarrow_x E$

$\dashv\vdash$ β -reduction $D \equiv \forall \vec{x}. F, E \equiv \forall \vec{x}. G, F \vdash y \prec z$ and $F \rightarrow_z G$ is an axiom

n so

$$\begin{aligned} D & \\ & \equiv v\vec{x}. (F, G) \\ & \equiv v\vec{x}. (F \end{aligned}$$

Eqn

$$\equiv \text{vy}\bar{w} \cdot (G', H) \quad (\text{VMIG}) \text{ n } (\text{VSWAP})$$

n

$$\begin{aligned} & \text{v}\bar{w} \cdot (G', H) \\ & \rightarrow_c \text{v}\bar{w} \cdot (G', I) \\ & \equiv E' \end{aligned} \quad \begin{array}{l} \text{Eqn} \\ \text{Eqn} \end{array} \Downarrow$$

• or v

$$I \equiv \text{vy} \cdot I' \quad E' \equiv \text{v}\bar{w} \cdot (G, I')$$

so n s s o u t t o u $\text{H} \rightarrow_c I$ n t t t t o n
poss t s (BUILD) n s

$$\begin{aligned} D & \equiv \text{v}\bar{w} \cdot (G, z = \text{rec } F \text{ in } M) \\ E & \equiv \text{v}\bar{w} \cdot (G, \text{local } F \text{ in } z = M) \\ E' & \equiv \text{v}\bar{w} \cdot (G, I') \\ \text{vy} \cdot I' & \equiv \text{local } F \text{ in } z = M \end{aligned}$$

~B ropos t on -

$$D \equiv \text{v}\bar{y} \cdot (G, H) \quad E \equiv \text{v}\bar{y} \cdot (G, I) \quad H \mapsto_c I \text{ s n } \Downarrow$$

n ropos t ons

$$\begin{aligned} D' & \equiv \text{v}\bar{y} \cdot D'' \\ (D'', x = M) & \equiv (G, H) \\ E' & \equiv \text{v}\bar{y} \cdot E'' \\ (E'', x = M) & \equiv (G, I) \end{aligned} \Downarrow$$

n ropos t ons - n - t r

• v

$$G \equiv (G', x = M) \quad D'' \equiv (G', H) \quad E'' \equiv (G', I) \Downarrow$$

so or n N

$$\begin{aligned} D', x = N & \\ & \equiv (\text{v}\bar{y} \cdot D''), x = N \\ & \equiv (\text{v}\bar{y} \cdot (G', H)), x = N \\ & \mapsto_c (\text{v}\bar{y} \cdot (G', I)), x = N \\ & \equiv (\text{v}\bar{y} \cdot E''), x = N \\ & \equiv E', x = N \end{aligned} \quad \begin{array}{l} \text{Eqn} \\ \text{Eqn} \\ \text{Eqn} \\ \text{Eqn} \\ \text{Eqn} \end{array} \Downarrow$$

• or v

$$H \equiv (H', x = M) \quad D'' \equiv (G, H') \quad I \equiv (I', x = M) \quad E'' \equiv (G, I')$$

n s n s s o u $\text{H} \rightarrow_c I$ n t t

o $G, H \rightarrow_x G, I$ n so $D \rightarrow_x E^-$
o For n N, $H', x = N \rightarrow I', x = N$,
n so $D', x = N \rightarrow E', x = N^-$

-B ropos t on

$$D \equiv \text{v}\bar{x} \cdot F \quad E \equiv \text{v}\bar{x} \cdot G \quad F \vdash y < z \quad F \rightarrow_z G \text{ s n } \Downarrow$$

n n o n v r t s o t t x x n ropos t on t r

• v

$$\bar{x} = \bar{y}\bar{w}\bar{z} \quad D' \equiv \text{v}\bar{y}\bar{z} \cdot [x/w]F[x/w] \Downarrow$$

n so

$$\begin{aligned} E & \\ & \equiv \text{v}\bar{x} \cdot G \\ & \equiv \text{v}\bar{y}\bar{w}\bar{z} \cdot G \\ & \equiv \text{v}\bar{y}\bar{w}\bar{z} \cdot [x/w]G[x/w] \end{aligned} \quad \begin{array}{l} \text{Eqn} \\ \text{Eqn} \\ \text{Eqn} \end{array} \Downarrow$$

$$[x/w]F[x/w] \rightarrow_z [x/w]G[x/w]$$

ropos t on

$$[x/w]F[x/w]$$

- $D \equiv D$

so $(\nabla \text{IND})_x D \rightarrow_x E_x$ n so $D \rightarrow_x \rightarrow_c F^-$

- $D \rightarrow_x E_x$ n so $D \rightarrow_x \rightarrow_c F^-$

(IND) s s r-

(VIND) s s r-

□

➤ For closed D if x is tagged in D and $D \rightarrow_c^* E$ then $D \rightarrow_x^* \rightarrow_{\neg x}^* E$

➤ F^- t $D \rightarrow_c^n E_x$ n pro n u t on on n^-

- $n =$ t n $D \equiv E$ so $D \rightarrow_x^* \rightarrow_{\neg x}^* E^-$

- $n >$ t n $D \rightarrow_c F \rightarrow_c^n E_x$ n ropos t on x s t n F
so n u t on $F \rightarrow_x^* \rightarrow_{\neg x}^* E_x$ so ropos t on $D \rightarrow_x^* \rightarrow_c \rightarrow_{\neg x}^* E_x$ n so
 $D \rightarrow_x^* \rightarrow_{\neg x}^* E^-$ □

➤ For closed D if x is tagged in D

$$\begin{aligned} & \gamma \vec{v} \cdot (I, \mathcal{V}(wvG) \cdot G) \\ & \equiv \vec{v} \cdot H \\ & \equiv E \end{aligned}$$

Eqn γ
Eqn γ
 \square

us $D \rightarrow_x \gamma E^-$

For closed D if $D \rightarrow E$ then $D \Downarrow_x$ iff $E \Downarrow_x$

F^-

$$\begin{aligned} \Rightarrow & \text{ } \bullet D \rightarrow_c E, \text{ t n } \text{ ropos t on } \Downarrow_x, E \Downarrow_x^- \\ & \bullet D \rightarrow_\gamma E, \text{ t n } \text{ ropos t ons } \text{ n } \Downarrow_x^- \\ & \text{tag}_x D \rightarrow_x \dots \rightarrow_x F \\ & \downarrow \gamma \\ & \text{tag}_x E \end{aligned}$$

- $\text{tr } s \ F_i = (x_i = M_i)$, $n \ w_i = \varepsilon^-$
- For i su t $D[\bar{x}/\bar{z}] \vdash x \sim x_i \ (x_i = M_i[\bar{x}/\bar{z}]) \rightarrow_c \forall \bar{w}_i . F_i[\bar{x}/\bar{z}]$ n so $E \rightarrow_c^* F[\bar{x}/\bar{z}]^-$
- $r \ , \ D[\bar{y}/\bar{z}] \rightarrow_c^* F[\bar{y}/\bar{z}]^-$
- $t \mathcal{R} \ \forall \text{ss } D[\bar{x}/\bar{z}] \text{ s u t on su t } t \bar{x} \mathcal{R} \bar{y}^- \text{ n } t \mathcal{R}' \text{ t s}$
 $\text{str t on ont n n } \mathcal{R} \text{ su t } t \bar{w}_i \bar{w}_i \bar{w}_i \mathcal{R} \bar{w}_j \bar{w}_j \bar{w}_j^- \text{ n s o } \mathcal{R}' \text{ s}$
 $\forall \text{ss } (G, x = M, F, \dots, F_n) [\bar{x}/\bar{z}] \text{ s u t on, n so } F[\bar{x}/\bar{z}] \vdash \bar{x} \sim \bar{y}^- \quad \square$

n (VMIG)

$$\begin{aligned} & \forall \vec{x}. (D, \text{local } G \text{ in } x = M', \text{local } H \text{ in } y = N') \\ & \equiv \forall \vec{x}. \nu(\text{wv } G) . \nu(\text{wv } H) . (D, G, H, x = M', y = N') \end{aligned}$$

n for t $\frac{1}{2}$ n t on s u t on

$$\forall \vec{x}. \nu(\text{wv } G) . \nu(\text{wv } H) . (D, G, H, x = M', y = N') \vdash x \sim y$$

so n u t on

$$\forall \vec{x}. \nu(\text{wv } G) . \nu(\text{wv } H) . (D, G, H, x = \forall y, y = N') \Downarrow_z$$

n so

$$\begin{aligned} & \forall \vec{x}. (D, x = \forall y, y = N) \\ & \equiv \forall \vec{x}. (D, x = \forall y, y = \text{rec } H \text{ in } N') && \text{Eqn } \gamma \\ & \rightarrow \forall \vec{x}. (D, x = \forall y, \text{local } H \text{ in } y = N') && \text{B D} \\ & \quad \forall \vec{x}. (D, \text{local } G \text{ in } \varepsilon, x = \forall y, \text{local } H \text{ in } y = N') && \gamma \\ & \equiv \forall \vec{x}. \nu(\text{wv } G) . \nu(\text{wv } H) . (D, G, H, x = \forall y, y = N') && \text{VMIG} \end{aligned}$$

n so Equ t on n ropo s t on

$$\forall \vec{x}. (D, x = \forall y, y = N) \Downarrow_z$$



ot r s s r s r

$$\perp \circ f = \perp$$

is so un- or t

$$\text{fix}(\text{set } Xg) \circ f = \text{fix}(\text{set } Xg)$$

From the sets s to $s \circ$ on D it $\llbracket D \rrbracket = \llbracket D \rrbracket \circ f^{-1}$

$\neg(\text{wv}[\llbracket D \rrbracket] \subseteq \text{wv}D)$ An n u t on on D^{-}

$(\text{wv}[\llbracket D \rrbracket] \supseteq \text{wv}D) \checkmark$ $\text{wv}[\llbracket D \rrbracket] \text{wv}D$ t n ∇ $x \in \text{wv}D$ n $x \notin \text{wv}[\llbracket D \rrbracket]$ n

\top

$$= \text{read } x \circ (x = \top)$$

$$= \text{read } x \circ \llbracket D \rrbracket \circ (x = \top)$$

$$= \text{read } x \circ \llbracket$$

$$\text{roptn } \downarrow \\ x \notin \text{wv}[\llbracket D \rrbracket]$$

$$= \text{read } x \circ f \qquad f = g \circ f$$

↙ $x \notin X$ t n

$$\begin{aligned} & \text{read } x \circ (\text{set } Xg)^{n+} \perp \circ f \\ &= \text{read } x \circ (\text{set } Xg)((\text{set } Xg)^n \perp) \circ f \qquad \text{D } \leftarrow n \leftarrow f^n \\ &= \text{read } x \circ f \qquad \text{ropn } \rightarrow \end{aligned}$$

↘ us $(\text{set } Xg)^{n+} \perp \circ f \leq f^-$

us

$$\begin{aligned} f &= g \circ f \\ &\Rightarrow \bigvee \{ (\text{set } Xg)^n \perp \circ f \mid n \text{ in } \omega \} \leq f \qquad \text{A } \circ \vee \\ &\Rightarrow \bigvee \{ (\text{set } Xg)^n \perp \mid n \text{ in } \omega \} \circ f \leq f \qquad \text{ } \circ \text{ s } \text{ ont nuous} \\ &\Rightarrow \text{fix}(\text{set } Xg) \circ f \leq f \qquad \text{D } \leftarrow n \leftarrow \text{fix} \end{aligned}$$

For ↙ p ↘ $\text{wv } f = X, \text{wv } g = Y \text{ n } X \cap Y = \emptyset \text{ t n } \vee \text{ p rt } \downarrow$

$$\text{fix}(\text{set}(X \cup Y)(f \circ g)) = f \circ \text{fix}(\text{set}(X \cup Y)(f \circ g))$$

n so t $\circ \vee$

$$\text{fix}(\text{set } Xf) \circ \text{fix}(\text{set}(X \cup Y)(f \circ g)) \leq \text{fix}(\text{set}(X \cup Y)(f \circ g))$$

r

$$\text{fix}(\text{set } Yg) \circ \text{fix}(\text{set}(X \cup Y)(f \circ g)) \leq \text{fix}(\text{set}(X \cup Y)(f \circ g))$$

us

$$\begin{aligned} & \text{set}(X \cup Y)(\text{fix}(\text{set } Xf) \circ \text{fix}(\text{set } Yg)(\text{fix}(\text{set}(X \cup Y)(f \circ g)))) \\ &= \text{fix}(\text{set } Xf) \circ \text{fix}(\text{set } Yg) \circ \text{fix}(\text{set}(X \cup Y)(f \circ g)) \qquad \text{ropn } - \\ &\leq \text{fix}(\text{set } Xf) \circ \text{fix}(\text{set}(X \cup Y)(f \circ g)) \qquad \text{Eqn} \\ &\leq \text{fix}(\text{set}(X \cup Y)(f \circ g)) \qquad \text{Eqn} \end{aligned}$$

• $x \in \text{wv}D t \quad n$

[[$\text{rec}D$ in M

$$= [[D]]$$

- Assu

$(M) (w \psi \rightarrow \chi)$

\downarrow_w so proposition -

$(D, w = M, x = M) \downarrow_x$

in $(z = x \ y) \sqsubseteq E \sqsupseteq (D, w = M, x = M)$ t r

of $z = x$, so $M = x \ y$, so $(D, w = M, x = M) \uparrow_x$ s on

• \uparrow_x on π \uparrow_x π \uparrow_x F su t t

$(F, w = \dots) = E \dots (E) \dots$

• F^-

- An intuition on ϕ^- on \mathcal{F}^n intuition $\phi = \psi \rightarrow \chi^-$

$\Rightarrow \mathcal{F} \models D(x \psi \rightarrow \chi)$ intuition $D \downarrow_x$ so intuition $v w. D \downarrow_x$ For intuition
 $(z = x \ y) \sqsubseteq E \sqsupseteq (v w. D)$, intuition v intuition s , intuition χ , intuition \mathcal{F}^n
 $F \sqsupseteq (z = x \ y)$ intuition t

$$E \equiv v v. F \quad F \sqsupseteq [v/w] D[v/w]$$

so intuition

$$\models [v/w] D[v/w] (x \psi \rightarrow \chi) \quad \mathcal{J}$$

intuition so

$$\models E (y \psi) \\ \Rightarrow \models v v. F (y \psi)$$

- $w = x \text{ t n}$ $\vdash_{\text{fn}} \text{r s } \bar{y} \text{ n I s u t t}$
 $H \equiv v\bar{x}\bar{y}. (F, G, I, w = M, z = w \ y)$
 so $t\bar{w} = wvG, \text{ n t v n } \bar{v} \text{ r s } \bar{\text{---}} \text{ n s n } \models D (x \ \psi \rightarrow \chi),$
 ropos t on

$$v\bar{x}. (F, v = \text{rec } G \text{ in } M)[v/w] \ (v \ \psi \rightarrow \chi)$$

n, ro t $\vdash_{\text{fn}} \text{t on } \subseteq$

$$(z = v \ y)$$

$$\subseteq v\bar{x}. (F[v/w], G, I, v = (\text{rec } G \text{ in } M)[v/w],$$

$$w = M[v/w], z = v \ y)$$

$$\supseteq v\bar{x}. (F, v = \text{rec } G \text{ in } M)[v/w]$$

n

$$\models H (y \ \psi)$$

$$\Rightarrow \models v\bar{x}\bar{y}. (F, G, I, w = M, z = w \ y) (y \ \psi) \quad \text{Eqn } \checkmark$$

$$\Rightarrow \models (F, G, I, w = M, z = w \ y) (y \ \psi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models (F, G, I, [v\bar{v}/w\bar{w}]G[v\bar{v}/w\bar{w}],$$

$$v = M, w = M, z = w \ y) (y \ \psi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models (F[v/w], G, I, [v\bar{v}/w\bar{w}]G[v\bar{v}/w\bar{w}],$$

$$v = M[v/w], w = M[v/w], z = v \ y) (y \ \psi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models (F[v/w], G, I, v = (\text{rec } G \text{ in } M)[v/w],$$

$$w = M[v/w], z = v \ y) (y \ \psi) \quad \text{n n}$$

$$\Rightarrow \models (F[v/w], G, I, v = (\text{rec } G \text{ in } M)[v/w],$$

$$w = M[v/w], z = v \ y) (z \ \chi) \quad \text{Eqns } \text{ n}$$

$$\Rightarrow \models H (z \ \chi) \quad \text{r}$$

$$\text{us } \models E (x \ \psi \rightarrow \chi)^-$$

- $x \neq w \neq z \text{ t n t proc s s } \text{r}^-$

(OTHER) $D \rightarrow_c E$ s pro_v t out B D t n n s o t t

$$D \subseteq D' \text{ p s } D' \rightarrow_c E' \supseteq E$$

$$E \subseteq E' \text{ p s } D \subseteq D' \rightarrow_c E'$$

n $\models D (x \ \psi \rightarrow \chi) \text{ t n } D \downarrow_x \text{ so ropos t on } \downarrow_x, E \downarrow_x \text{ n or}$

n $(z = x \ y) \subseteq F \supseteq E, \text{ n } \vdash_{\text{fn}} G \text{ s u t t}$

$$F \equiv (G, z = x \ y)$$

n t w r s, so

$$(w = x \ y) \subseteq (G, w = x \ y, z = x \ y) \supseteq E$$

n $\vdash_{\text{fn}} H \supseteq D \text{ s u t t}$

$$H \rightarrow_c F$$

n

$$\models F (y \ \psi)$$

$$\Rightarrow \models (G, z = x \ y) (y \ \psi) \quad \text{Eqn}$$

$$\Rightarrow \models (G, z = x \ y, w = x \ y) (y \ \psi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models (H, w = x \ y) (y \ \psi) \quad \text{n n}$$

$$\Rightarrow \models (H, w = x \ y) (w \ \chi) \quad \models D (x \ \psi \rightarrow \chi)$$

$$\Rightarrow \models (G, z = x \ y, w = x \ y) (w \ \chi) \quad \text{n n}$$

$$\Rightarrow \models (G, z = x \ y, w = x \ y) (z \ \chi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models (G, z = x \ y) (z \ \chi) \quad \text{roprn } \checkmark$$

$$\Rightarrow \models F (z \ \chi) \quad \text{Eqn}$$

us or n $(z = x \ y) \subseteq F \supseteq E$

$$\models F (y \ \psi) \Rightarrow \models F (z \ \chi)$$

$$\text{sq } \models E (x \ \psi \rightarrow \chi)^-$$

ot r r t on s s o n s r -

□

3.11 Full abstraction

nt ss t on, so t tt o D s u str t or on urr nt p r u t on s n s t t on urr nt p r u t on s t s s u str t o s t ost out r ostr u t on, n so on urr nt p r u t on s t t s o p u t t on p o r s t ost out r ostr u t on s p r o o s t s s t r u t u r s t on -

- so t t $\Gamma \vdash D \ \Delta \ll [\Delta] \leq [D][\Gamma]$, t us s o t t p r o c s s t s s o u n n o p t o r t n o t t o n s n t s s s r o p o s t o n t p r u t o n q u v n t r o p o s t o n -

- t n s o t $\Gamma \vdash D \ \Delta \text{ t n } \Gamma \models D \ \Delta, \text{ n t t } \Gamma \models D \ \Delta \text{ t n } [\Delta] \leq [D][\Gamma]^- \text{ u s t t r p r s n t t o n s o t o r q u v n t s s r o p o s t o n t p r u t o n q u v n t r o p o s t o n -}$

- F n, so t t u str t on s n p r o v n a t t r o p r s n t t o n s t o q u v n t s s r o p o s t o n t p r u t o n q u v n t r o p o s t o n -

us, AB A, Y n G s t n q u s n p t t o p r u t o n -

$$\Gamma \vdash M \ \phi \text{ iff } [\phi] \leq [M][\Gamma]^-$$

$$\Gamma \vdash D \ \Delta \text{ iff } [\Delta] \leq [D][\Gamma]^-$$

• F^-

$D E \Rightarrow$ $\text{sound } \dashv\text{For } \phi, \text{ to } \text{pro}_{\mathcal{V}}(x), \llbracket \Delta \rrbracket \leq \llbracket x = M \rrbracket[\Gamma] \text{ n } \llbracket \phi \rrbracket \leq \llbracket M \rrbracket[\Delta]$
 $t \text{ n}$

$$\begin{aligned} & \llbracket x \ \phi \rrbracket \\ & \leq (x = \llbracket M \rrbracket[\Delta]) \llbracket \Delta \rrbracket && \text{H pot } \leq \\ & \leq (x = \llbracket M \rrbracket[\Gamma]) (\llbracket x = M \rrbracket[\Gamma]) && \text{H pot } \leq \\ & = \llbracket x = M \rrbracket[\Gamma] && \text{ropn } \end{aligned}$$

•

ot r s s r s r

C • $\dashv\text{E E E} \Leftarrow$ An n u t on on M n $D\text{-For } \phi, x \neq y$ n

$$\llbracket \phi \rrbracket \leq \llbracket x \neq y \rrbracket[\Gamma]$$

t n t r $\llbracket \phi \rrbracket = \perp$, so $\vdash \phi = \omega$ n so $\Gamma \vdash x \neq y \ \phi$, or

$$\begin{aligned} & \llbracket \phi \rrbracket \leq \llbracket x \neq y \rrbracket[\Gamma] \\ & \Rightarrow \llbracket \phi \rrbracket \leq \text{apply}[\Gamma(x)] \llbracket \Gamma(y) \rrbracket && \text{D } \llcorner \text{n } \llcorner \llbracket x \neq y \rrbracket \\ & \Rightarrow \llbracket \Gamma(y) \rightarrow \phi \rrbracket \leq \llbracket \Gamma(x) \rrbracket && \text{ropn } \dashv \\ & \Rightarrow \vdash \Gamma(x) \leq \Gamma(y) \rightarrow \phi && \text{ropn} \\ & \Rightarrow \vdash \Gamma \leq x \ \Gamma(y) \rightarrow \phi, y \ \Gamma(y) && \text{D } \llcorner \text{n } \llcorner \leq \\ & \Rightarrow \Gamma \vdash x \neq y \ \phi && (\leq) \dashv \end{aligned}$$

$$(z = x \ y) \sqsubseteq E \sqsupseteq (D, x = \lambda w.M)$$

- $\models D(x \phi \rightarrow \psi)$ then $D \downarrow_x$ so Corollary $\not\models [D] \sigma x \neq \perp \neg A$ so, or
 \models s y n z
 true
 \Rightarrow

4 Conclusions

nt sp pr, v n_v st t t r t ons p t nt s nt not on
full abstraction n t p nt t ont n qu *concurrent graph reduction*
v s o n t t

- Con urr nt p r u t on n s p op r t on pr s nt t on
nt st BEY n B D s *chemical abstract machine*, n
E s *polyadic π calculus*
- t n qu s AB A Y n G s *lazy λ calculus* n

urs v r tons, o v r sor s- AD H so n, st t st
r tons p t n p r u t on n t D_∞ o t unt p λ u us
s BA E DEG, or or t s, top s tr p up
E E n AB A Y n G -
BA A DEG et al_ r s r o or on term graph rewriting,
ntro u BA A DEG et al_, n sur_ E A AY et al_
n t ot r p p rs n EE et al_s EE et al_, oo - r
p s r v r s r to r tons, ut r root, n, o s s B - / A -

λ H HA A A D EA A - A not r ppro to t op r t on s
 nt s or p r u t on s λ H HA A n EA A s AZY
 CF HA t n s s CF t let r t on s s
 λ n st p op r t on s nt s or n our s nt

$$(\text{let } D \text{ in } M) \Downarrow (\text{let } E \text{ in } N)$$

- λ s s nt s s s r to ours n A CHB Y s pt t t
- AZY CF HA s t p n s onstru tors n onstru
 tors or oo ns n n tur nu rs
- n let pr ss ons r n us r t r t n rec pr ss ons t s n
 t s or points os so s r n or t on

$$(\text{let } D \text{ in let } x = (\mu x. M) \text{ in } M) \Downarrow (\text{let } E \text{ in } N)$$

Functores prod e sum são monóides no Cat .
 O produto cartesiano prod é um monóide com o objeto pt e o morfismo id .
 A soma sum é um monóide com o objeto pt e o morfismo id .

O produto cartesiano prod é um monóide com o objeto pt e o morfismo id .
 A soma sum é um monóide com o objeto pt e o morfismo id .
 O produto cartesiano prod é um monóide com o objeto pt e o morfismo id .
 A soma sum é um monóide com o objeto pt e o morfismo id .

$\text{fst} : T \times U \rightarrow T$ e $\text{snd} : T \times U \rightarrow U$

O produto cartesiano prod é um monóide com o objeto pt e o morfismo id .

$$M = \dots | \text{pair } xy | \text{fst } x | \text{snd } x$$

O produto cartesiano prod é um monóide com o objeto pt e o morfismo id .

$\perp, |$ or r^- choose λ to t λ u s t r u r s v
 r t o n s s

$$M = \dots | \text{choose } xy$$

$$D = \dots | o = \perp | o = | | o = r$$

t t o p r t o n s n t s λ v n

$$x = \text{choose } yz, y = ?M \mapsto x = \text{choose } yz, y = M$$

$$x = \text{choose } yz, z = ?M \mapsto x = \text{choose } yz, z = M$$

$$o = \perp, x = \text{choose } yz, y = \lambda w. M \mapsto o = |, x = \text{choose } yz, y = \lambda w. M$$

$$o = \perp, x = \text{choose } yz, z = \lambda w. M$$

E D - *Combinator Graph Reduction: A Congruence and its Applications* - t s s, or n v r s t -

AC A E, - *Categories for the Working Mathematician* - Gr u t s n, t t s -
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E - *Functional Programming: An Introduction to the Theory and Practice* -

E - *Communication and Concurrency* - r nt H -

E - *Lambda Calculus* - po π u us tutor - n Proc. International Summer School on Logic and Algebra of Specification, r to r o -

Y G F, A - *Abstract Interpretation and Optimising Transformations for Applicative Programs* - Dt s s, E n ur s n v r s t D pt Co put r n -

G, C-H - *The Lazy Lambda Calculus: An Investigation into the Foundations of Functional Programming* - Dt s s, p r Co s, on n v r s t -

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H HA, A, - n EA, A, - *An Introduction to the Theory of Computation* - n Proc. ESOP -

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 C -

AD H, C - *Semantics and Pragmatics of the Lambda Calculus* - Dt s s, or n v r s t -

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 str t r t on $\partial[[D]]$,
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 t or,
 ω_{CPOE} ,
 ω_{CPO} ,
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 $\text{t } C_{\perp}$,
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 $\text{pr ss } D_{\Gamma}$,
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 o $\forall \bar{x}. D$,

o $\forall x. D$,
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 t no $x = M$,
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 $\text{t n } (\cdot)_{\perp}$,
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 $\text{fv } M$,
 $\text{r } \text{ o t on } D \rightarrow_{\forall} E$,
 l,

█
E ,
s u t on,
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