ont nts o C pt rs n 4 r t r su t o o or t v r s r w t sup rv sor tt w H nn ss – r r n r v rs ons o t s two pt rs v n pu s s uss

Con n

n o on

1-1 u p ss n ro st ---------------------------------- 8
1- o n ----------------------------------
1- n qu po nt n u t on ----------------------------------
1-4 rv w o t t s s ----------------------------------
1-4-1 r r qu s t s ---------------------------------- 1

o p n o B on

-1 r ns t on s st s ---------------------------------- 11
- D t o ns ---------------------------------- 11
--1 orp s s o t n u s ---------------------------------- 1
-- E p s o t o ns ---------------------------------- 1
-- or out t nv ron nts ---------------------------------- 1
- or r p s ---------------------------------- 1
--1 orp s s o s or r p s ---------------------------------- 14
-- or r p s to tr ns t on s st s ---------------------------------- 1
-- or r p s ov r 1 ---------------------------------- 18
-4 u p ss n CC ---------------------------------- 1
- B s u t on ---------------------------------- 1
--1 or s u t on ----------------------------------
8 - or r p s w t ss nt ---------------------------------- 4

n q [illegible] po n n on
1 A str t ons n FI — [illegible]
1.1 proo s st [illegible] — [illegible]
– oun n ss n o p t n ss or stron s u t on — [illegible]
– w s — [illegible]
4 Co p t n ss or o s rv t on on ru n — [illegible]
– r s — [illegible]
– twor s o r u r pro ss s — [illegible]
–1 r t r s vs– np r t r s r t ons — [illegible]
– E p — [illegible]

C [illegible] p q p p C p p o p n
1 u r — [illegible]

O

C pt r p n n s ———————————————————————— 1

D n n t on r t on un tor ———————————————————— 1
– E r p r t on s nt s or v u p ss n CC —————————— 1
– or op r t on s nt s o CC ——————————————— 1
– t p r t on s nt s or v u p ss n CC – ———————— 1

p r t on s nt s or os nts———————————————
– In r n u s ———————————————————————
– In r n u s ———————————————————————
– A str t op r t on s nt s ——————————————
– tt rn str t op r t on s nt s —————————— 4
– E p ns on ws or CB p r ————————————— 4

Gr r or t o ———————————————————
– Int rpr t t on o o – ——————————————————
– o o n ru s– ——————————————————
– F po nt ru s———————————————————————
– Int rpr t t on o rst or r po nt o – ————————
– s t onstru t on– ————————————————————
– n r t p rs r wr t n r t on ———————————
– or nt rpr t t ons o or u ——————————
– t onstru t on or s or s nt s ———————————
D n t on o un t on ***DApps*** ov r or u ————————————
F ow r or pro ss ——————————————————
or r p or ————————————————————

In r n ru s or n t CC ——————————————— 1
– w n r n ru s or onst nts n str t ons ———————— 1
– E p ns on ws or p r n r str t on ——————————— 1
– p nt t on o ***Spec*** ——————————————————— 1 1

C p

n o on

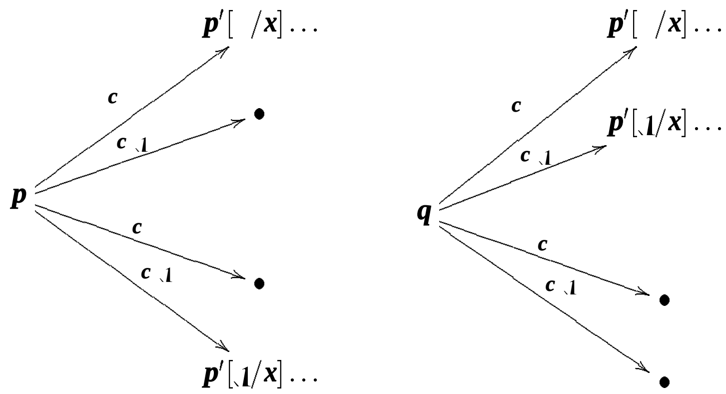
Co un t on n on urr n r t two un nt on pts w r us to o n un rst n o p n s st s– A r s st n r s s o t on o n p rts n us utono ous nts o out t r us n ss n p n nt n so t nts t n on urr nt o not o s st s w – In p rt u r t to ptur t un v our o t s st v w o st – v o to pt t t t s sp t o s st s n o w nt r t on or o un t on– s s t s t n t o on urr n t or or pro ss u us t t s v n r s to r o o wor ov r t st w s– rst t or o on urr n t ons r to t t or o tr n ts – s o s ros s n r s t on o ut t n w t ons or v nts p r o on urr nt ut w t w s so ow s nt ro t n t o ppro w s n ppr t on o t stru tur o r s st s n p rt u r t o s o pon nts nt r t n to o w o – s ons r t ons w r to r o n n t r rs n t w o t stu o pro ss s– n n p n nt t or s o pro ss s w r v op roun t o o un t n nts p r o n to n v s t ons t u pr s nt n r t ru ts o t ns v r s r nto pro ss s or p – s u prov s nt t s r pt ons o o un t n nts n nt so *pure* o s o t s n u s o un t on s o so s n rons t on so t t t t t tu t tr ns tt ro on nt to not r s str t w – s s p r v str t on or t or t purpos s ut or n sp t ons on w s s to r t n rt n un nt sp ts o o un t on– For p on w s to s r proto o s w r tu ss s r n s nt tw n nts n utur v our p n s upon t ont nt o su ss s– n u s w n orpor t t s or 4 4 1 1 – 4 1 – 1 – u 1 4 – 1

s ros r t to tr ns t on s st s quot nt s u t on qu v n [ ]. B s r t s o n u t v n n proo s o qu v n n n w tn ss n s u t on r t ons prov n nt pr o o n u t v r son n . A sur o t su ss o s u t on s t t tr ns t ons st s n s u t on qu v n r now so n us s onv n nt o o n o n u t v proo t n qu or s qu nt o put t on n t un t on s tt n [ ].

v n s r n w t s r rr to s *strong* s u t on qu v n . r s n t u o rs r not on o s u t on qu v n *weak* s u t on. st n u s n prop rt o w s u t on s t t o un t on s no on r tr t s n o s rv t on. In CC t s s o not n t t o o un t on p t us n u s o τ tron s u t on t s τ t ons nto ount n ps tr o nt rn o put t ons w r s w s u t on tr ts t s s uno s rv n str ts ro t . ons r s u t on s qu v n s us v nt s t ss n r r t r r to [ ] or tr nt o t s nt s o t st n qu v n or v u p ss n n u s.

n t r u t t tr ns t ons st s r p r t qu t to o t tu v our o s st s. r ppropr t n ss s s to nt r t to ptur t r n n stru tur o pro ss s w s r t nt r st n p rt o pro ss v our n t on urr nt s tt n . How v r n or r to o v u p ss n pro ss s tr ns t ons st w n to nvo n r. u qu tous n o n n t stru tur nt r t o t pro ss s o pro s . A propos or o n v u p ss n pro ss s r t n so sort o rst or r tr ns t on s st s oun n [4]. H nn ss n n propos *symbolic graphs* s su t n r s t on o tr ns t on s st s.

us t t *symbolic* to r pr s nt t t t s nt t ppro to t s opt n or r to u our r p s. us v u s n v u pr ss ons r un nt rpr t n r r pr s nt so t r un t on s o s. t t s n n on t ons r *syntactic graphs* to pt t rn t v n ors o r p s. pr r to us t t s o to r t ts us n t wor o Bur t [ ]. or n r us o t wor *symbolic* n [ ] r rs to n n n ppropr t r pr s nt t on o pro s st t sp r t r t n wor n w t t st t sp p t . For p us n BDDs to r pr s nt tr ns t on r t ons n o u s or o n nt o μ u us [ ] s n nst n o s o ppro . urs o r p s prov s o r pr s nt t on o t n rs .



It s r to s t t t tr ns t ons st o s o *p* n *q* r nt st n r not on o s u t on or v u p ss n pro ss s t t on o t ns v n r s n o n nto pur n u s n opt n r s or n not on o s u t on] or t r su t n tr ns t on s st s s *early* s u t on]– It s no surpr s t t *p* n *q* r to s r– r s n pt t rn t v n t on o s us su qu st on p n su t t on t o n o v u s us n t pr ss v t o t t pr ss ons ow – or t s or p rt n s u t ons o s o r p s n [4] nt t r su t t v n n u s o t s t tt tt wor s p r tr w t r sp t to t o ns t t n s t t v r t ons r r t v to t – rou out t t s s w w o s on us t t r s soun n ss or o p t n ss w t out t ***relative***

t t r t ons us or n n r urs v pro ss s un qu v ours– prop rt o s u t on qu v n w pro t s t t v n r t on

$$X \Longleftarrow E$$

n two pro ss s p n q su t t p s s r to $E[p/X]$ n q s s r to $E[q/X]$ t n p n q ust t s v s s r– s prop rt o s not o or r tr r pro ss s r p t ons E ut s u r nt or u r s r pt ons w r o urr n s o X n E r w t n t s op o n t on pr – s prop rt o s or pro ss s w t r sp t to tr qu v n 1 1 – It s s own n 1 4 t t t s proo t n qu on on w t ru s or r ov n un u r n ss s su nt or r t r s n s u t on qu v n or r u r pro ss t s– s s unsurpr s n s t un qu po nt n u t on proo ru s s to ptur t o n u t v vour o s r t p r t – pr n p n pr ss s n n r n ru

$$\frac{\vdash p = E[p/X]}{\vdash p = X}$$

w r $X \Longleftarrow E$ s u r t s t u r n ss t t nsur s t soun n ss o t s ru – A su

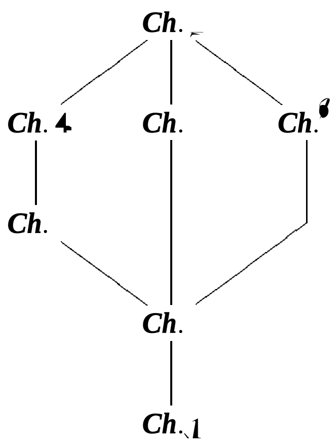


Figure 1.1. C pt r p n n s

ntu t v n r s t on o t un qu po nt n u t on ru w s r v ro H n n ss n 4 n.s propos ru n us to t t r str t on on p r t rs– s ow r t v o p t n ss w t r sp t to stron s u t on or u r r u r pro ss s–E t n n t s wor urt r w o on to r t r s o s rv t on on ru n n s ov rt t t r r ws u to n r 4 n st us to str t ro nt rn t ons–A s uss on on t r t ons p tw n p r t r s t on n p r o pos t on or v u p ss n n u s s v n n w on u t pt r w t n p qu v n proo –

n t t s s w t s ort pt r st t n our on us ons n v nu s or utur r s r –

A t ou w r v w t s n t ons o tr ns t ons st s s u t on n v u p ss n CC r t w t pur pro ss u wou st n t v nt n r n t st s s– r r t r r to t t t oo s 4 1 or oo ntro u t on to t su t–Issu s r t n to v u p ss n s nt s r p n n u n no pr or p r n w t su n u s s r qu r – prop rt o rst or r μ u us pr s nt n C pt r s s on t o μ u us u to o n n r tt 1 n qu nt n w

o p o n

G v n two t n u s $\boldsymbol{D} = (\boldsymbol{Var}, \Sigma_{\boldsymbol{V}}, \boldsymbol{I}, \boldsymbol{Pr})$ n $\boldsymbol{D}' = (\boldsymbol{Var}', \Sigma_{\boldsymbol{V}'}, \boldsymbol{I}', \boldsymbol{Pr}')$ t s s ns to s w t p

$$\frac{\boldsymbol{n} \overset{\boldsymbol{b},\tau}{\longmapsto} \boldsymbol{n}'}{[\boldsymbol{n},\delta] \overset{\tau}{\longrightarrow} [\boldsymbol{n}',\delta]} \qquad \delta \models \boldsymbol{b}$$

$$\frac{\boldsymbol{n} \overset{\boldsymbol{b},\boldsymbol{c}\ \boldsymbol{e}}{\longmapsto} \boldsymbol{n}'}{[\boldsymbol{n},\delta] \overset{\boldsymbol{c}\ \boldsymbol{v}}{\longrightarrow} [\boldsymbol{n}',\delta]} \qquad \delta \models \boldsymbol{b}, \boldsymbol{v} = [\![\boldsymbol{e}]\!]\delta$$

$$\frac{\boldsymbol{n} \overset{\boldsymbol{b},\boldsymbol{c}\ \boldsymbol{x}}{\longmapsto} \boldsymbol{n}'}{[\boldsymbol{n},\delta] \overset{\boldsymbol{c}\ \boldsymbol{v}}{\longrightarrow} [\boldsymbol{n}',\delta[\boldsymbol{v}/\boldsymbol{x}]]} \quad \delta$$

$$\frac{}{\tau.p \xrightarrow{\tau} p} \qquad \frac{}{c\;e.p \xrightarrow{c\;[\![e]\!]} p} \qquad \frac{\forall v \in Val}{c\;x.t \xrightarrow{c\;v} t[v/x]}$$

$$\frac{p \xrightarrow{\alpha} p'}{b \rightarrow p \xrightarrow{\alpha} p}$$

ro ss s ***p***

o s ow p q w ust n t n p rt t ons n ov s or o t tr ns t ons ro ot p n q– s r t t s ort tr ns t on $p \overset{,c\,x}{\longmapsto} p'$– t w r qu r s p rt t on su t t or oo n b n t s p rt t on w now t t b u r nt s $c\,x$ tr ns t on ro q to no w s t st b r t to p'– p rt t on w r qu r t n s $\{even(x), odd(x)\}$–I w t $even(x)$ w s t t $q \overset{,c\,x}{\longmapsto} q''$ w t $p' \;^{even(x)}\; q''$ n or $odd(x)$ w us $q \overset{,c\,x}{\longmapsto} q'$ w t $p' \;^{odd(x)}\; q'$– o urt r p rt t on n s r qu r n w not t tt oun v r x o urs n t p rt t on n t s n r t rt n t s u t on–

pr v ous su st t ts o s u t on ou t to pr s rv on r t on– s ounts to s n t t

$$t \quad u \text{ p } s\, t\delta \quad u\delta \text{ or } \delta.$$

r t ons p tw n on r t n s o s u t on s n t u t t r–

opo on *t ^{b} u if and only if $t\delta$ $u\delta$ for all $\delta \models b$.*

oo [4] ■

o p n n

A s ort o n o s o r p s s t t ou t r oo t o n n n t r n n pro ss s n t t r r st n v u p ss n pro ss s w r ntu t v n t n stru tur ut r o n n t s o r p s–For p t pro ss $X( )$ r

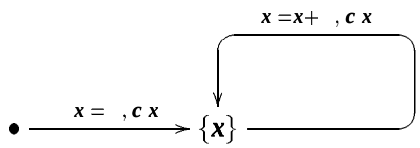
$$X \Longleftarrow \lambda x.c\,x.X(x+\ )$$

r p t outputs t s qu n o v n nu rs on c– stru tur o t s pro ss s v r s p t r s s n ***state*** ro w t r s c output tr ns t on– tu tr ns t on r p w so pp ns to t s o r p ort s pro ss oo s

$$X(\ ) \xrightarrow{c} X(\ ) \xrightarrow{c} X(4) \xrightarrow{c\ 4} \cdots$$

An n n t r p s n us to o r r t v s p stru tur –

n ppro to r t n t s stu t on w s pr s nt n p n nt n n ows [ ]– r w s so s r so ut on pr s r n [ ] us n r nt o s to s o r p s– n n ows s so ut on nvo v ntro u n p t ss nts or su st tut ons nto t r s o t r p s– us r u r v ours su s t p $X( )$ n s r t r tr ns t ons on w t su st tut on to s r ow t t p rt o t pro ss s t tr ns t on– s o r p w t ss nt or $X( )$ now oo s



or n r s o r p w t ss nt s s o r p w os s r now w t tr p (b, θ, α) w r $b \in \boldsymbol{BoolExp}$ $\alpha \in \boldsymbol{Act}$ n θ s n ss nt $x = e$– wr t $m \overset{b,\theta,\alpha}{\longmapsto} n$ to not r s o t r p n s t t $fv(b,e) \subseteq fv(m)$ $fv(\alpha) \subseteq x$ n $fv(n) \subseteq x \cup bv(\alpha)$–

o r p s w t ss nt n un o nto s o r p s n s r nn r to s tur t n s o r p w t su st tut ons– t r t n us n s p su st tut ons ow v r w s tur t w t r tr r su st tut ons– us w n r t s o r p rou t ru

$$\frac{m \overset{b,\theta,\alpha}{\longmapsto} n}{(m,\sigma) \overset{b\sigma,\alpha\theta\sigma}{\longmapsto} (n,\theta\sigma)}$$

s ows us to n s or s u t on ov r r p s w t ss nts s p s n t t t r r sp t v s or r p s s r–

An port nt tur o r p s w t ss nts s t t n t o r u r v u p ss n CC t t s t su n u w t out p r o pos t on n r str t on s o ***finite*** r p w t ss nt– s t s pro t n [ ] n or r to pr s nt n or t or r u n t t o stron s u t on own to n v t o oo n pr ss ons out t – p rt u r n u t t t s oo n pr ss ons r s r n s ss nt rst or r pr t o w t p r t r s po nts–

In C pt r w pr s nt proo s st or r son n out r u r v u p ss n CC pro ss s– on t us s or r p s w t ss nt p It n t ou on t

C p p

on B on o C o
B o n

turn to t wor o ro st n s st s or our rst onstr t on o t s or t n qu – n u w ons r s CB v u p ss n pro ss u us w r o un t on tw n nts s t t ro st n o v u s– n u s s r n st to v u p ss n CC ut s u t w s n ron s t on op r

D s r	Input	utput
$- \xrightarrow{w} -$		
$\dfrac{w \notin S}{x \in S\ \ t \xrightarrow{w} x \in S\ \ t}$	$\dfrac{v \in S}{x \in S\ \ t \xrightarrow{v} t[v/x]}$	
$e\ \ p \xrightarrow{w} e\ \ p$		$\dfrac{[\![e]\!] = w}{e\ \ p \xrightarrow{w} p}$
$\dfrac{\forall i \in I \cdot p_i \xrightarrow{w} p_i}{\sum_I p_i \xrightarrow{w} \sum_I p_i}$	$\dfrac{\exists i \in I \cdot p_i \xrightarrow{v} p'}{\sum_I p_i \xrightarrow{v} p'}$	$\exists i \in$

E I $\dfrac{}{p = p}$ $\dfrac{p = q}{q = p}$ $\dfrac{p = q \quad q = r}{p = r}$

AXI $\dfrac{p = q \in \text{A} \quad \text{s}}{p = q}$

C G $\dfrac{p_1 = q_1 \quad p = q}{p_1 + p = q_1 + q}$

α C $\dfrac{}{x\ t = y\ t[y/x]}$ $y \notin fv(t)$

I $\dfrac{\sum_{i \in I} \tau\ t_i[v/x] = \sum_{j \in J} \tau\ u_j[v/x] \quad \text{or v r } v \in Val}{\sum_{i \in I} x\ t_i = \sum_{j \in J} x\ u_j}$

$\dfrac{p = q, [[e]] = [[e']]}{e\ p = e'\ q}$

B $\dfrac{[[b]] =}{b \gg p = p}$ $\dfrac{[[b]] =}{b \gg p =}$

Figure 3.2. In r n s

v u pro ss p s ons r to nt prov n t t t utur v our o p o s not p n upon t r pt o t v u – s ns or pr t t

$$v\ p + x\ v\ p \quad {}_n\, v\ p$$

or n pro ss p– ot t t p s os so x o s not o ur r n $v\ p$ n t r or nnot t t utur v our o t t $v\ p$–In q s n pro ss w n s r *i.e.* $q \longrightarrow$ t n

$$q + x\ q \quad {}_n\, q$$

us q n s r n v u – s n turn ns t t

$$w\ (q + x\ q) \simeq_n w\ q.$$

$\boldsymbol{x}\ \boldsymbol{t} = \boldsymbol{x}\ \boldsymbol{u}$ ro[redacted] t pot s s $\boldsymbol{t} = \boldsymbol{u}$

E I

$$\frac{}{\triangleright \boldsymbol{t} = \boldsymbol{t}} \qquad \frac{\boldsymbol{b} \triangleright \boldsymbol{t} = \boldsymbol{u}}{\boldsymbol{b} \triangleright \boldsymbol{u} = \boldsymbol{t}} \qquad \frac{\boldsymbol{b} \triangleright \boldsymbol{t} = \boldsymbol{u} \quad \boldsymbol{b} \triangleright \boldsymbol{u} = \boldsymbol{v}}{\boldsymbol{b} \triangleright \boldsymbol{t} = \boldsymbol{v}}$$

AXI

$$\frac{\boldsymbol{t} = \boldsymbol{u} \in \mathrm{A \quad o\ s}}{\triangleright \boldsymbol{t} = \boldsymbol{u}}$$

C G

$$\frac{\boldsymbol{b} \triangleright \boldsymbol{t}_1 = \boldsymbol{u}_1 \quad \boldsymbol{b} \triangleright \boldsymbol{t} \ = \boldsymbol{u}}{\boldsymbol{b} \triangleright \boldsymbol{t}_1 + \boldsymbol{t} \ = \boldsymbol{u}_1 + \boldsymbol{u}}$$

Proof. [4.1]. ∎

s n t o proo s st or op n t s w now s ow t t o s $\mathcal{A}$ on w t n r s t on o o s *cl-Noisy* to op n t s prov soun n o p t o t s t on or stron no s on ru n ov r $\mathcal{SA}$. n r s t on o *cl-Noisy* s

$$\boxed{\textit{Noisy} \quad e\,(t + x\, t) = e\, t \qquad x \notin fv(t)}$$

w r t s t o t o

$$\sum_{i \in I} b_i \gg e_i\, t_i.$$

ot t t n os nst nt t on o su t s r s v r tr ns tt v u s n t n not r v n nput. A ow n s t us o not t on t us n us $\mathcal{A}_{\mathcal{N}}$ to r r to t o s $\mathcal{A}$ on w t t n r s o *Noisy*. so wr t $\mathcal{A}_{\mathcal{N}} \vdash b \triangleright t = u$ to n t t $b \triangleright t = u$ n r v n t proo s st o F ur ro t o s n $\mathcal{A}_{\mathcal{N}}$.

Lemma *(Axiom Noisy is sound) For all δ, if $x \notin fv(t)$ then $(e\,(t + x\, t))\delta \simeq_n (e\, t)\delta$.*

Proof. Cons r n r tr r os nst nt t on o *Noisy* $w\,(p + x\, p) \simeq_n w\, p$ n p s t o p. It s su nt to s ow t t $p + x\, p \sim_n p$. t I t nt t r t on ov r nts. s ow t t $I' = I \cup \{(p + x\, p, p)\}$ s

oo As n [4] w us ropos t on – to prov t t w n v r s no s s or s u t on t n

$$R \stackrel{def}{=} \left\{ (t\delta, u\delta) \mid \exists b \cdot \delta \models b \;\; n \;\; (t,u) \in S^b \right\}$$

s no s s u t on– r w n v r $\mathcal{R}$ s no s s u t on t n

$$S^b_{\mathcal{R}} \stackrel{def}{=} \{(t,u) \mid \delta \models b \;\; p \;\; s\,(t\delta, u\delta) \in \mathcal{R}\}$$

o s no s s or s u t on– r su t o ows s r ro t s– s ou not r t t t proo o t s t or n s r t pr ss v n ss o t oo n t n u –Es s nt w r qu r t pow r to s r v n s ts o nv ro nts– s ssu s s uss n [4]– ■

A t p prop rt on r qu r s n or r to prov o p t n ss o proo s st s t t to tr ns o t s nto rt n s nt t o s– us o two t p s o t s s nt t o s st n r o s w ow us to so t t n v u su n s o t n no o s w or t t r n s s o t oo n u r s ont n w t n t –

F rst t *t* s s to n ***stan ar or*** t s o t o

$$\sum_{i \in I} b_i \gg e_i\; t_i + \sum_{i \in I} \quad r \quad not$$

s n CA E n ropos t on – – w n o t n or K

$$\vdash c_K \rhd t = \sum_K c_K \gg (\sum_{k \in K} \alpha_k . t_k).$$

us v n t t $\bigvee c_K =$ CA E v s

$$\vdash \rhd t = \sum_K c_K \gg (\sum_{k \in K} \alpha_k . t_k).$$

It s r t t t pt o t t r t s un n s r su t o t s tr ns o t ons– ■

As n p o ow t utu us v u r s o no r o s n us u w r r t r r to [4] ropos t on – n not t t t proo t r n so us to on u t t

$$\mathrm{I} \gg \quad \frac{b \rhd \sum_{i\in I} c_i \gg \tau\ t_i = \sum_{j\in J} d_j \gg \tau\ u_j}{b \rhd \sum_{i\in I} c_i \gg x\ t_i = \sum_{j\in J} d_j \gg x\ u_j}$$

w r $x \notin fv(b, c_i, d_j)$ s r v ru o t proo s st –

G v n st n r o $t \equiv \sum_{i\in I} b_i \gg \alpha_i . t_i$ w not t t w n v oo n on t ons to s r w n t n nput or s r –For nst n w now t t oo n b w u r nt t t t s t t to r v v u $b \models \bigvee_I$

t or or n t CC t r s w t r sp t to w s u t on on ru n $\approx_c$ 1 4 p 1
r son s r r r t ons p tw n w s u t on $\approx$ n s u t on on ru n $\approx_c$
$p \approx q$

uppos t t $u + x$ $u \xrightarrow{d_j, e} u_{jl}$– n s or w us t t t t t $_n^{b''}$ u to t t n p rt t on n ov – uppos t t $u + x$ $u \xrightarrow{d, x} u'$–B ssu pt on d s d_j or –C r d nnot d_j us s w v r st r s $u \not\xrightarrow{d_j, x} u'$– us d ust n u' ust u–A n b'' p rt t ons ts r to t t t n ov $t \xrightarrow{c_i, x} t_{ik}$–

C r sts $u \xrightarrow{d_j, x} u'$ su t t or r $t \xrightarrow{c_i, x} t_{ik}$ $u' \not\simeq_n^{b''} t_{ik}$– tr r u nt o C s on w r s $t + x$ $t \simeq_n^{b''} u$ n $b' \models DC(t)$–

C p t r o t ov – t s or v r t $^{c_i, x}$

son t sn o

Empty $x \in \emptyset\ X = \_$

now t t $\boldsymbol{x} \in \boldsymbol{I}(\boldsymbol{q}) - \boldsymbol{I}(\boldsymbol{p})$ $\boldsymbol{p} \xrightarrow{\boldsymbol{v}} \boldsymbol{p}$ w n v r $\boldsymbol{v} \in \boldsymbol{I}(\boldsymbol{q}) - \boldsymbol{I}(\boldsymbol{p})$– o w r qu r t ro $\boldsymbol{q}$– $\boldsymbol{q} \xrightarrow{\boldsymbol{v}} \boldsymbol{q}'$ s $\boldsymbol{v} \in \boldsymbol{I}(\boldsymbol{q})$ n $\boldsymbol{v} \notin \boldsymbol{I}(\boldsymbol{p})$ so $\boldsymbol{p} \xrightarrow{\boldsymbol{v}} \boldsymbol{p}$– B us $\boldsymbol{p}$ $_{\boldsymbol{n}}\boldsymbol{q}$ w t n now t t $\boldsymbol{p}$ $_{\boldsymbol{n}}\boldsymbol{q}'$

o t us p rt u r $\boldsymbol{v} \in \boldsymbol{S}_l^j$ n s ow t s n n rr – now t t $\boldsymbol{v} \in \boldsymbol{S}_l$ n $\boldsymbol{t}_j[\boldsymbol{v}/\boldsymbol{x}]$ $_n$ $\boldsymbol{u}_l[\boldsymbol{v}/\boldsymbol{x}]$– For onv n n t $\boldsymbol{p}, \boldsymbol{q}$ not $\boldsymbol{t}_j[\boldsymbol{v}/\boldsymbol{x}$

r rn s no s ns to s w t *I*(, *t*) s our – ppro w t s to r t r s t oor n pr ss ons ***b*** or w ot r wor s $b \wedge b' \models \neg b''_j$ or j–G v n t s w n pp n u t on to o t n $I(t\delta) = I(t_1\delta) = I(b \wedge b', t_1)$–But t s s t s r r pt –H n $I(t\delta) = I(b, t) = \emptyset$–

o w ust ons r t s w r K s non pt –B un o t w ust v t t $b \models b'$–
s o ows us b_k n K s o t o $b' \wedge b''_k$ or so b''_k– In t s s b ust t_1 un o n n u t on v s

I

D s r	Input	utput
$\overset{,Val}{\longmapsto}$		
$x \in S\ t \overset{,Val\setminus S}{\longmapsto} x \in S\ t$	$\dfrac{}{x \in S\ t \overset{,x\in S}{\longmapsto} t}$	
$e\ t \overset{,Val}{\longmapsto} e\ t$		$e\ t \overset{,e}{\longmapsto} t$
$\dfrac{t \overset{b,S}{\longmapsto} t \quad u \overset{b',S'}{\longmapsto} u}{t+u \overset{b'\wedge b,S\cap S'}{\longmapsto} t+u}$	$\dfrac{t \overset{b,x\in S}{\longmapsto} t'}{t+u \overset{b,x\in S}{\longmapsto} t'}$	$\dfrac{t \overset{b,e}{\longmapsto} t'}{t+u \overset{b,e}{\longmapsto} t'}$
$b' \gg t \overset{\neg b',Val}{\longmapsto} b' \gg t$		
$\dfrac{t \overset{b,S}{\longmapsto} t}{b' \gg t \overset{b,S}{\longmapsto} b' \gg t}$	$\dfrac{t \overset{b,x\in S}{\longmapsto} t'}{b' \gg t \overset{b'\wedge b,x\in S}{\longmapsto} t'}$	$\dfrac{t \overset{b,e}{\longmapsto} t'}{b' \gg t \overset{b'\wedge b,e}{\longmapsto} t'}$

Figure 3.5. tt rn str t op r t on s nt s

t t ns on s ons rv t v on – tr ns t ons r t ons r s or r w t oo n v u s t n s u r s– r n s o ur n tr ns t ons o t o $\overset{b,x\in S}{\longmapsto}$ now or t w t t p tt rn nput n $\overset{b,S}{\longmapsto}$ w r S r or s t s t o v u s w s r – ur n t ov s to pr s nt t not on o *patt rn no sy sy bo c b s u at on* w t s nto ount t t t n n t n tr ns t on or $t \overset{b,x\in S}{\longmapsto} t'$ o s so n oo n wor $b \wedge x \in S$– t s to s t t t t x

$\boldsymbol{t} \overset{\boldsymbol{b}_1, x \in \boldsymbol{S}}{\longmapsto} \boldsymbol{t}'$ t r sts v r $\boldsymbol{z}$ su t t $\boldsymbol{z} \notin \boldsymbol{fv}(\boldsymbol{b}, \boldsymbol{t}, \boldsymbol{u})$ n $\boldsymbol{b} \wedge \boldsymbol{b}_1 \wedge \boldsymbol{z} \in \boldsymbol{S}$ p rt t on $\boldsymbol{B}$ su t t or $\boldsymbol{b}' \in \boldsymbol{B}$ t r sts $\boldsymbol{u} \overset{\boldsymbol{b}\ ,y \in \boldsymbol{S}'}{\longmapsto} \boldsymbol{u}'$ su t t $\boldsymbol{b}' \models \boldsymbol{b}$, $\boldsymbol{b}' \models \boldsymbol{z} \in \boldsymbol{S}'$ n $\boldsymbol{t}'[\boldsymbol{z}/\boldsymbol{x}]$ $\overset{\boldsymbol{b}'}{}_{\boldsymbol{pn}}$ $\boldsymbol{u}'[\boldsymbol{z}/\boldsymbol{y}]$

A n s [redacted] tr on t ons on $\boldsymbol{u}$

4- $S \neq \emptyset, S' \neq \emptyset$–

B or – –‗ t r sts t', u' su t t t_i $\overset{b''}{\underset{pn}{}}$ t' n u_j $\overset{b''}{\underset{pn}{}}$ u' n $d(t') < d(t$

r t n $S_K \stackrel{def}{=} \bigcap_{k \in K} (Val - S_k)$ t

$$
\begin{aligned}
Exp(t \mid u) \;=\; & \sum_{i\in I, j\in J} (c_i \wedge d_j \wedge e_i \in S_j) \gg e_i\ (t_i \mid u_j[e_i/x]) \\
+ & \sum_{i\in I, j\in J} (c_i \wedge d_j \wedge e_j \in S_i) \gg e_j\ (t_i[e_j/x] \mid u_j) \\
+ & \sum_{i\in I, K\ J} (c_i \wedge \bigwedge_{k\in K} \neg d_k \wedge e_i \in S_{J-K}) \gg e_i\ (t_i \mid u) \\
+ & \sum_{j\in J, K\ I} (\bigwedge_{k\in K} \neg c_k \wedge d_j \wedge e_j \in S_{I-K}) \gg e_j\ (t \mid u_j) \\
+ & \sum_{i\in I, j\in J} (c_i \wedge d_j) \gg x \in S_i \cap S_j\ (t_i \mid u_j) \\
+ & \sum_{i\in I, K\ J} (c_i \wedge \bigwedge_{k\in K} \neg d_k) \gg x \in (S_i \cap S_{J-K})\ (t_i \mid u) \\
+ & \sum_{j\in J, K\ I} (\bigwedge_{k\in K} \neg c_k \wedge d_j) \gg x \in (S_j \cap S_{I-K})\ (t \mid u_j).
\end{aligned}
$$

Figure 3.6. E p ns on ws or CB p r

---

oo s n s prov r t ro t op r t on s nt s– ■

o o t t tr ns t on un t ons w us t o ow n o n n n u t v on t s– s o n r qu r s t t w t n t s n tur o t t n w t t un t on s o s us to pr ss t f n g tr ns t on un t ons– t $g^{-1}(S) = \{v \in Val \mid g(v) \in S\}$– ot t t $\tau \notin g^{-1}(S)$ s $\tau \notin S$ n g s str t– us Λ to not un t on ro Var to tr ns t on un t ons n w t $e\Lambda$ not t su st tut on $e[g(x)/x \mid x \in fv(e), g = \Lambda(x)]$–

$\langle \cdot \rangle_{(f,g,\Lambda)} = \cdot$

$\langle e\ t \rangle_{(f,g,\Lambda)} = f(e\Lambda)\ \langle t \rangle_{(f,g,\Lambda)}$

$\langle x \in S\ t \rangle_{(f,g,\Lambda)} = x \in g^{-1}(S)\ \langle t \rangle_{(f,g,\Lambda[g/x])}$

$\langle b \gg t \rangle_{(f,g,\Lambda)} = b\Lambda \gg \langle t \rangle_{(f,g,\Lambda)}$

$\langle \sum_{i\in I} t_i \rangle_{(f,g,\Lambda)} = \sum_{i\in I} \langle t_i \rangle_{(f,g,\Lambda)}$

$\langle t_{(f',g')} \rangle_{(f,g,\Lambda)} = \langle t \rangle_{(f \cdot f', g' \cdot g, \Lambda)}$

r s to nsur t t n ro st tr ns ss on ro $\langle t \rangle_{(f,g,\Lambda)}$ s tr ns t us n t un t on f– n pro ss $p_{(f,g)}$ pp rs to r v v u v w p t t ont nu n pro ss to so t n o t o $p'[v/x]$ ut w s t op r t on s nt s F ur –1 t t t pro ss $p_{(f,g)}$ w st pp r n to r v v tu r v s t v u $g(v)$ n ont nu s to v $p'[g(v)/x]$– o ptur t s v our n t o n w n to r or us n t Λ un t on t w tr ns t on g w s us w n x un oun n t n su s qu nt us t t tr ns t on w r v r x o urs–

p If $\Lambda(x) = Id$ then $\langle t \rangle_{(f,g,\Lambda[h/x])}\delta[v/x] \equiv \langle t \rangle_{(f,g,\Lambda)}\delta[h(v)/x]$.

oo tru tur n u t on on t–t nt r st n s s r w n t s pr t –
uppos t s $e\ t'$–t n

$$\langle t \rangle_{(f,g,\Lambda[h/x])}\delta[v/x] \equiv f(e\Lambda[h/x]\delta[v/x])\ \langle t$$

- $\boldsymbol{p} \downarrow \boldsymbol{v}$ t n $\boldsymbol{q} \stackrel{\varepsilon}{\Longrightarrow} \boldsymbol{q}'$ or so[redacted] $\boldsymbol{q}'$ su t t $\boldsymbol{q}' \downarrow \boldsymbol{v}$
- $\boldsymbol{q} \downarrow \boldsymbol{v}$ t n $\boldsymbol{p} \stackrel{\varepsilon}{\Longrightarrow} \boldsymbol{p}'$ or so[redacted] $\boldsymbol{p}'$

$$\alpha.(X + \tau.Y) + \alpha.Y =_{ccs} \alpha.(X + \tau.Y)–$$

$$X + \tau.X =_{ccs} \tau.X–$$

n ortun t t o v ous v rs ons o $T1$ n T or CB r not soun – v r s n or p t t p s not n n r w s r to τ p w p s t t v τ $p \neq v$ p– For T $p + \tau$ $p = \tau$ p w run nto u t s w n p s ow to r v v u v s –For t n τ p s r v ut $p + \tau$ p

$p \overset{w}{\Longrightarrow} p'$ pr s $p \overset{w}{\longrightarrow} p'$–

$v \in I(p)$ n $p \overset{v}{\Longrightarrow} p'$ pr s $p \overset{v}{\longrightarrow}\overset{\varepsilon}{\Longrightarrow} p'$–

$v \in I(p)$ n $p \overset{\tau v}{\Longrightarrow} p'$ pr s $p \overset{v}{\longrightarrow} p'$–

s on t on t t $v \in I(p)$ n s s n s our r w nt p t t t w n prov p on ru nt to s tur t v rs on o p– In or r to o t s w ust not ntro u n n w nput t ons nto t s tur t v rs on o p pu n v t on s out ro n τ t on– on t on $v \in I(p)$ u r nt s t t n nput t on ntro u w s r poss –

D v on ***For any standard form*** $p \in \mathcal{SPA}$, $p \overset{w}{\Longrightarrow} q$ ***implies*** $\mathcal{A}_{\mathcal{P}\tau} \vdash_{cl}$ $p = p + w\, q$.

oo B n u t on on t n t o t r v t on $p \overset{w}{\Longrightarrow} q$–

s s s str t orw r I $p \overset{w}{\longrightarrow} q$ t n $w\, q$ s su n o p us p s st n r – I pot n o + v s t proo $\mathcal{A}_{\mathcal{P}\tau} \vdash_{cl} p = p + w\, q$–

uppos t t $p \overset{w}{\longrightarrow} p' \overset{\tau}{\Longrightarrow} q$– n n uw ow w now t t $\mathcal{A}_{\mathcal{P}\tau} \vdash_{cl} \boldsymbol{p} = \boldsymbol{p} + \tau\, \boldsymbol{p}'$ t D r v t on – so now t t w n prov $\mathcal{A}_{\mathcal{P}\tau} \vdash_{cl} \boldsymbol{p}' = \boldsymbol{p}' + \boldsymbol{x} \in \boldsymbol{S}\ \boldsymbol{q}'$ or so s t $\boldsymbol{S}$ n so t $\boldsymbol{q}'$ su t t $\boldsymbol{v} \in \boldsymbol{S}$ n $\boldsymbol{q}'_{\boldsymbol{v}}$ s $\boldsymbol{q}'[\boldsymbol{v}/\boldsymbol{x}]$– Co n n t s v s

$$\mathcal{A}_{\mathcal{P}\tau} \vdash_{cl} \boldsymbol{p} = \boldsymbol{p} + \tau\, (\boldsymbol{p}' + \boldsymbol{x} \in \boldsymbol{S}\ \boldsymbol{q}').$$

ow o ***Tau3*** wou pp $\boldsymbol{S} \subseteq \boldsymbol{I}(\boldsymbol{p})$ ut w nnot nsur t s– How v r us n t

- uppos t r sts p^τ su t t $p \xrightarrow{\tau} p^\tau$ n or q' su t t $q \stackrel{\tau}{\Longrightarrow} q'$ w v $p^\tau \not\approx q'$–In t s s w s ow t t or s–

rst not t t

$$\begin{aligned} I(p + x \in S \;\; p) &= I(p) \cup I(x \in S \;\; p) \\ &= I(p) \cup (I(q) \setminus I(p \end{aligned}$$

$\backslash 1 - \boldsymbol{U} = \emptyset$

H r w v $\boldsymbol{p} = \boldsymbol{q} + \boldsymbol{x} \in \boldsymbol{V}$ $\boldsymbol{q} + \tau\ \boldsymbol{q}$

oo s s s p tt r o n t t o n ru s soun w t r sp t to t

$t \xmapsto{b_1, e} t'$ t r sts $b \wedge b_1$ p rt t on B n or $b' \in B$ t r sts $u \stackrel{b, e'}{\Longrightarrow}_p u'$ su t t $b' \models b$, $b' \models e = e'$ n $t' \approx^{b'} u'$

$t \xmapsto{b_1, x \in S} t'$ t r sts v r z su t t $z \notin fv(b, T, U)$ n $b \wedge$

A n w ssu , w t out oss o n r t, t t t s st n r o – now t t $t \overset{b',\tau\, S'}{\Longrightarrow} t'$ so suppos

$$t \overset{b_1,\tau}{\Longrightarrow} u \overset{b\ ,S'}{\longmapsto} u \overset{b\ ,\varepsilon}{\Longrightarrow} t'$$

w r $b' = b_1 \wedge b \ \wedge b$ – uppos so t t $u \ \equiv \sum_I \ b_i \gg x \in S_i \ u_i$– n

$$b \ = \bigwedge_{j \in J} \neg b_j \quad \text{n} \quad S' = \bigcap_{j \in I \ \setminus J} (Val \setminus S_j)$$

or so s r n $J \subseteq I$ – t $B_u = \{b \wedge b_K \mid K \subseteq I\ \}$ u un o b p rt t on n o s rv t t w n v r $j \in K \cap J$ w v t t $b \wedge b_K \models b_j$ n $b \wedge b_K \models b \ \models \neg b_j$– n t s ontr pos t v w v t t $b \wedge b_K \neq$ p s $K \cap J = \emptyset$–

ur nt nt on s to prov

$$\mathcal{A}_{\mathcal{P}\tau} \vdash b \wedge b_K \rhd \tau\ u = \tau\ (u + x \in S\ \ u)$$

pp n o ***P-Noisy*** or AB D w n $b \wedge b_K =$ to u or $b \wedge b_K$–In or r to o t s w n to s ow t t $S \cap I(b \wedge b_K, u) = \emptyset$ w n v r $b \wedge b_K \neq$ –

uppos t n t t $b \wedge b_K \neq$ n suppos or ontr t on t t $v \in S \cap I(b \wedge b_K, u)$– s ns t t $v \in S$ n $v \in S_j$ or so $j \ \in K$– But $v \in S \subseteq S'$ p s t t $v \in S' = \bigcap_{j \in I \ \setminus J} (Val \setminus S_j)$ t t s $v \notin S_j$ or $j \in I \ \setminus J$– r or $j \ \notin I \ \setminus J$ n w on u t t j

or $\quad \boldsymbol{b_u} \in \boldsymbol{B_u}$

o t n t r sult us n ***P-Noisy*** n ***Tau1*** – Assu t n t t S s not pt – nnot ppl n u t on t l us t o nt pt s o t t s s not r s – How v r t D o pos t on or 4 – v s t s t'' n u'' su t t $d(t'') < d(t')$ $d(u'') < d(u')$ $t'' \approx^{b''} t'$ n $u'' \approx^{b''} u'$ – t out loss o n r l t w ssu t t $d(t') \leq d(u')$ – B n u t on t ollows t t $\mathcal{A}_{\mathcal{P}\tau} \vdash b'' \rhd \tau\ t' = \tau\ t''$ w n $\mathcal{A}_{\mathcal{P}\tau} \vdash b'' \rhd z \in S\ t' = z \in S\ t''$ I – It s l r t t

$$t' + x \in S\ t'' =^{b''} u' + x \in S'\ u' + \tau\ u'$$

n n u t on s ppl r r l n

$$\mathcal{A}_{\mathcal{P}\tau} \vdash b'' \rhd t' + x \in S\ t'' = u' + x \in S'\ u' + \tau\ u'.$$

s n t pr v ous r sult w n su st tut t' or t'' n ppl A n ***P-Noisy*** to t

$$\mathcal{A}_{\mathcal{P}\tau} \vdash b'' \rhd \tau\ t' = \tau\ (u' + x \in S'\ u' + \tau\ u').$$

t r sult ollows s n t s w r S s pt – Appl t on o CA E n I pot n w l now l

$$\mathcal{A}_{\mathcal{P}\tau} \vdash b_u \rhd \tau\ t' + \tau\ u' = \tau\ u'.$$

■

s n s s our pl t n ss proo – r sult n l t to op w t n t CB us n t o n s o t on 4 n t t s w s t proo s st s or stron no s on ru n – s prov s CB w t pow r ul qu t on l t or o o s rv t on on ru n – l t t t on ru n w ons r w s r v r r s ul t ons us n n l s nt s or CB w s to v n l t t l t s nt s nt r l – r or w n t s C pt r w t so nts out l t s ul t ons n CB –

A n o CB

ons r w t t l t s nt s or CB t n r u t t t o not oo put t on l s ns n t s p r – r l r C pt r t t ov to l t s nt s nvolv r n up r pt on $c\ x.t \xrightarrow{c\ v} t[v/x]$ nto two p rts F rst w ons r t ov

$$c\ x.t \xrightarrow{c} (x)t$$

to str t on t t s un t on r ***Val***

s nt s s not t r r. r son or t s s t t t o un t on proto o s r v n nts.'r t ons to ***values*** n o r . nt ***p*** n ***q*** r spon to t v u 1 r v n n s r n r sp t v ut t r spon to t v u ot r v n . pro r s t t t t s nt s or ***p*** w nt to tr t t v u s 1 n n on o o v u s. r s s n nput ov ro ***p*** t s n

$$p \xrightarrow{\{1,\ \}} (x \in \{1,\ \})t,$$

w n s s n r t on ro ***q*** ut o ours ***q*** nnot prov s n r t on to t s tr ns t on. For t s r son w ons r t ut w r n vous o CB to unsu t to support t s nt s n o not pursu t s ssu n urt r.

C p

o o C n o n o

so or o p r t r s t on–I w t s p r t r z w wou v t or u

$$\nu X.\langle a\ x\rangle(x = z + \ ) \wedge X(x/z).$$

n t n nst nt t t s po nt. s p r t r to v n w r *B* s so oo n on t on on t v r s *t* s t no o s o r p n *F* s or u o rst or r *μ* u us w t r t v r s–

ustr t ow w n r s t proo s st tt w t t o ow n p –Cons r t po nt or u

$$A \equiv \nu X.\langle a\ x \rangle (x = z\ mod\ ) \wedge X.(z \oplus\ 1/$$

w r A'' s s A' ut w t ($z =$ 1, t) so n t t s t– n n w pp ru u st us n t oo n $z =$ – ot t t $z =$ 1 $\models$ ($z =$ [$z \oplus$ 1/z])– s s to t u nt

$$z = \quad \vdash t \quad A''$$

$$F \;=\; B \mid F \vee F \mid F \wedge F \mid \langle\tau\rangle F \mid [\tau]F \mid \langle c\;x\rangle F \mid [c\;x]F \mid \langle c\;\rangle G \mid [c\;]G \mid A.(e/x)$$

$$G \;=\; \exists x.F \mid \forall x.F$$

$$A \;=\; X \mid \nu X[\mathcal{A}]F \mid \mu X[\mathcal{A}]F$$

Figure 5.1. Gr r or t ro

n s nt t t or o o or ur w G r n s ov r qu nt or ur – o op r tors $\langle c\;x\rangle$ n $[c\;x]$ t s n rs or t v r x s o t qu nt rs $\exists x$ n $\forall x$ wr t $fv(F)$ or t r t v r s o F– s not on s r or or ur w t out po nts–F n A r n s ov r ***fixpoint abstractions*** ut t s on us to n o or ur t onstru t on $A.(e/x)$– s not s t ppr t on o t str t on A to t su st tut on $[e/x]$– s onstru t on n s o t r t v r s o x n A t us w v t t

$$fv(A.(e/x)) = fv(e) \cup (fv(A) \setminus \{x\})$$

w r $fv(e)$ s v n t t o n n $fv(A)$ s n

$$fv(\nu X[\mathcal{A}]F) = fv(\mu X[\mathcal{A}]F) = fv(\mathcal{A}) \cup fv(F) \quad \text{n} \quad fv(X) = \emptyset.$$

n t on o $fv(\mathcal{A})$ s t o v ous on – A or ur F s ***recursion closed*** $FV(F)$ s pt n s ***data closed*** $fv(F)$ s pt – I A s o t or $\nu X[\mathcal{A}]F$ or $\mu X[\mathcal{A}]$

on propos n [4] w r t s s own to r t r st or t s u t on qu v n – s t t t po nts prov no tr st n u s n pow r ov r pro ss s–

opo on *t* $\sim^b_L$ *u if and only if for all recursion closed formulae F with empty tag sets,*

$$t \models_b F \text{ iff } u \models_b F$$

oo uppos $\delta \models b$ n t p, q not $[t, \delta]$ n $[u, \delta]$ r sp t v – *if* r t on s prov n [4] us n t or u su to st n u s non s r pro ss s– s ow t onv rs – uppos $p \sim_L u$– n to s ow $p \in [\![F]\!]\rho\delta$ $q \in [\![F]\!]\rho\delta$– u t s r s n t s s o po nt or u – nnot w t po nts r t ut t s su nt to s ow t t t r su t or s or t r or n unw n n s–It s w nown t t $[\![\mu X.F]\!]\rho\delta = \bigcup_\alpha [\![\mu^\alpha X.F]\!]\rho\delta$ [1] w r t μ or u nnot t w t n or n r nt rpr t s

$$
\begin{aligned}
[\![\mu X.F]\!]\rho\delta &= \emptyset \\
[\![\mu^{\alpha+1} X.F]\!]\rho\delta &= [\![F[\mu^\alpha X.F/X]]\!]\rho\delta \\
[\![\mu^\gamma X.F]\!]\rho\delta &= \bigcup_{\alpha<}
\end{aligned}
$$

$$\textit{Id}\quad \frac{}{B \vdash t \quad B}$$

$$\textit{Case}\quad \frac{B_1 \vdash t \quad F, \ldots, B_n \vdash t \quad F}{\bigvee_{1 \le i \le n} B_i \vdash t \quad F}$$

$$\textit{Cons}\quad \frac{B_1 \vdash t \quad F}{B \quad \vdash t \quad F} \quad (B \quad \models B_1)$$

$$\textit{Ex}\quad \frac{B \vdash t \quad F}{\exists x.B \vdash t \quad F} \quad (x \notin fv(t,F))$$

$$\alpha\quad \frac{B \vdash t' \quad F'}{B \vdash t \quad F} \quad (t' \equiv t, F' \equiv F)$$

$$\wedge\quad \frac{B \vdash t \quad F_1 \quad B \vdash t \quad F}{B \vdash t \quad F_1 \wedge F}$$

$$\vee_L\quad \frac{B \vdash t \quad F_1}{B \vdash t \quad F_1 \vee F}$$

$$\vee_R\quad \frac{B \vdash t \quad F}{B \vdash t \quad F_1 \vee F}$$

$$\langle\tau\rangle\quad \frac{B \vdash t' \quad F}{B \wedge b \vdash t \quad \langle\tau\rangle F} \quad t \overset{b,\tau}{\longmapsto} t'$$

$$[\tau]\quad \frac{B \wedge b_1 \vdash t_1 \quad F, \ldots, B \wedge b_n \vdash t_n \quad F}{B \vdash t \quad [\tau] F}$$

$$\text{w} \quad \text{r} \quad \{(b_1,t_1), \ldots, (b_n,t_n)\} = \{(b,t') \mid t \overset{b,\tau}{\longmapsto} t'\}$$

$$\langle c \quad \rangle\quad \frac{B \vdash t' \quad F[e/x]}{B \wedge b \vdash t \quad \langle c \; x \rangle F} \quad t \overset{b,c \; e}{\longmapsto} t'$$

$$[c \quad]\quad \frac{B \wedge b_1 \vdash t_1 \quad F[e_1/x], \ldots, B \wedge b_n \vdash t_n \quad F[e_n/x]}{B \vdash t \quad [c \; x] F}$$

$$\text{w} \quad \text{r} \quad \{(b_1,t_1,e_1), \ldots, (b_n,t_n,e_n)\} = \{(b,t',e) \mid t \overset{b,c \; e}{\longmapsto} t'\}$$

$$\langle c \quad \rangle\quad \frac{B \vdash (y)t' \quad G}{B \wedge b \vdash t \quad \langle c \quad \rangle G} \quad (t \overset{b,c}{\longmapsto} (y)t')$$

$$[c \quad]\quad B \wedge b_1 \vdash (y_1)t_1 \quad F, \ldots, B$$

u st $B \vdash t \quad A.(z/z)$

4. t ***Val*** t n tur nu rs n t t r p $\mathcal{G}$ v two no s $\boldsymbol{t}_1, \boldsymbol{t}$ w t n $\boldsymbol{t}_1 \stackrel{a\ x}{\longmapsto} \boldsymbol{t}$ – str t on $\boldsymbol{\mu X}[\emptyset]\boldsymbol{F}$ w r $\boldsymbol{F}$ s $(\langle \boldsymbol{a}\ \boldsymbol{y}\rangle$

$$
\begin{aligned}
\boldsymbol{t}\mathrm{s}\ \mathrm{t}\boldsymbol{B} &= \boldsymbol{B} \\
\boldsymbol{t}\mathrm{s}\ \mathrm{t}\boldsymbol{F}_{1} \wedge \boldsymbol{F} &= \boldsymbol{t}\mathrm{s}\ \mathrm{t}\boldsymbol{F}_{1} \wedge \boldsymbol{t}\mathrm{s}\ \mathrm{t}\boldsymbol{F} \\
\boldsymbol{t}\mathrm{s}\ \mathrm{t}\boldsymbol{F}_{1} \vee &
\end{aligned}
$$

s t– s α onv rs on pr s two ro s n t s onstru t on rst o ur n t s tu t on w r t t s t o A_j ***decreases*** n s p ss n ro o u F_n to F_{n+1}– s ow t t t on w t s n pp n s $A_i \sqsubset A_j$–

s s s t tr to st s – ntro u t o n w t ***pointers***– po nt rs r s n so t t A_j n $F_n \in \mathcal{C}$ po nts to so $A_{j'}$ t $F_{n'}$ n $\mathcal{C}'$ or to so $A_{j'}$ n F – n n t po nt rs so t t A_i n F po nt to t st su o u A_j o F su t t $A_j \sqsubset A_i$ w t t su o u F tu po nt n to ts – u s qu nt F_n s po nt rs t n F_{n+1} n r ts t po nt rs ro F_n n n t s w r F_n s $\nu X[\mathcal{A}]F$ n F_{n+1} s $F[\nu X[\mathcal{A}^+]F/X]$ or so X w t o urr n o t n w r t su o u $\nu X[\mathcal{A}]F$ n F_{n+1} po nt to F_n– An nv r nt o t s po nt rs s t t A_i n F_n po nts to t o u A_j t $F_{n'}$ on $n' =$ n $A_j \sqsubset A_i$ or $n' \neq$ n $i = j$ n t t s t o A_i s r t r t nt t o A_j– s st s t s nv r nt t n t on o po nt rs t F – o s t t t nv r nt s nt n on $\mathcal{C}$ w oos so F_n or w t o s n ons r w $F_{n+1} \ll F_n$– I F_{n+1} s su o u o F_n t n n surv v n po nt rs r n r t n t nv r nt st o s– t rw s F_{n+1} s o t n un o n n w s t n w po nt rs s t s t s on p rt o t nv r nt–

su n our proo w suppos t n s su po nt rs ss n to t– n w n now s t t n or r or t t s t o A_j to ***decrease*** n p ss n ro F_n to F_{n+1} t nv r nt on po nt rs t s us t t v r o urr n o A_j n F_{n+1} ust po nt to A_i t F w t $A_i \sqsubset A_j$–

s prov s t t $A_i \sqsubset A_j$ w n v r $v^j_{n+1} > v^j_n$– But topo sort n t s us t t $i < j$ t us $v_{n+1} < v_n$ n t o r p or r n on v tors us $v^i_{n+1} < v^i_n$– s s tru or n so w v n n nt s n n n o v tors w t r sp t t

$$
\begin{array}{lllll}
(t, F_1 \wedge F\ ) & \rightarrowtail & (t, F_1) & \text{n} & (t, F_1 \wedge F\ ) \rightarrowtail (t, F\ ) \\
(t, F_1 \vee F\ ) & \rightarrowtail & (t, F_1) & \text{n} & (t, F_1 \vee F\ ) \rightarrowtail (t, F\ ) \\
(t, \langle\tau\rangle F & \rightarrowtail & (t', F) & \text{or} & t \overset{b,\tau}{\longmapsto} t' \\
(t, [\tau]F) & \rightarrowtail & (t', F) & \text{or} & t \overset{b,\tau}{\longmapsto} t' \\
(t, \langle c\ x\rangle F & \rightarrowtail & (t', F[e/x]) & \text{or} & t \overset{b,c\ e}{\longmapsto} t' \\
(t, [c\ x]F) & \rightarrowtail & (t', F[e/x]) & \text{or} & t \overset{b,c\ e}{\longmapsto} t' \\
(t, \langle c\ \rangle G) & \rightarrowtail & ((x)t', G) & \text{or} & t \overset{b,c\ x}{\longmapsto} t' \\
(t, [c\ ]G) & \rightarrowtail & ((x)t', G) & \text{or} & t \overset{b,c\ x}{\longmapsto} t' \\
((x)t, \forall y.F) & \rightarrowtail & (t[w/x], F[w/y]) & \text{w r} & w = new(fv((x)t, \forall y.F)) \\
((x)t, \exists y.F) & \rightarrowtail & (t[w/x], F[w/y]) & \text{w r} & w = new(fv((x)t, \forall y.F)) \\
(t, A.(e/z)) & \rightarrowtail & (t, A) & & \\
(t, \nu X[\mathcal{A}]F) & \rightarrowtail & (t, F[\nu X[\mathcal{A}^+]F/X]) & t \notin \mathcal{A} &
\end{array}
$$

w r $\mathcal{A}^+ = \mathcal{A} \cup (t$s t$\nu X[\mathcal{A}]F, t)$–

Figure 5.7. n r t p rs r wr t n r r t on

---

su t t F_i s o t or $\nu X_i[\mathcal{A}_i]F_i'$ n no po nt or ur r n ount r tw n F_i n F_{i+1}– n η_1 to [[t_1s tF_1]]/X_{k_1}]– u s qu nt w n η_{i+1} to η_i[[t_{i+1}s tF_{i+1}]]$\eta_i/X_{i+k_{i+1}}$]– w rs 4 4n I A – 1 s – 4 1 1 1– – I w

■

p p ***For finite $\mathcal{G}$ and pairs (t, F) generated from $(t\ \ , F\ \ )$ with η as above:***

$$[\![t\, sat F]\!]\eta \vdash t \quad F.$$

oo r r to t proo n **4** t ou w us w oun n u t on on o u o n r t p rs– on s s o nt r st r r pp t on n po nt o u –I ***t*** pp rs n t t s t o t po nt o u $\nu X[\mathcal{A}]F$ t n ru ν n t n t on o η v s t r su t– t rw s n u t on w now t t

$$[\![t s\ \ t F[\nu X[\mathcal{A}']F/X]]\!]\eta \vdash t \quad F[\nu X[\mathcal{A}']F/X]$$

w r $\mathcal{A}' = \mathcal{A} \cup (t s\ \ t \nu X[\mathcal{A}]F, t)$–But $[\![t s\ \ t F[\nu X[\mathcal{A}']F/X]]\!]\eta$ s s s n to $[\![t s\ \ t \nu X[\mathcal{A}]F]\!]\eta$ so ru ν_1 w v our r su t–

I ***F*** s t o u $A.(e/z)$ t n n u t on t s us tnn

(Completeness) For all formulae F with empty tag sets, finite $\mathcal{G}$, $fv(B) \subseteq fv(t)$,

$$t \models_B F \text{ implies } B \vdash t \quad F.$$

r n r o t s s t on s vot to st s n t s– s n n w s nt s t *symbolic semantics* or t o to t our p t n ss r su t– A s o nt rpr t t on t s r urs on nv ron nt n oo n pr ss on r t r t n t nv ron nt n or r to nt rpr t t r t v r s– oo n r pr s nts t s t o t nv ron nts w s t s t– It w us u to nt n t rou out t p t n ss proo v r str t o or t s oo n pr ss ons– ssu t t t v t o

$$B \wedge (z = e)$$

w r B s oo n pr ss on not ont n n n r urs on p r t rs n e s n t v tor o t pr ss ons so not ont n n n r urs on p r t rs– For not t on onv n n w s r su oo n s o ows– t ε r n ov r su st tut ons o t o

$$
\begin{aligned}
[\![B']\!]_s \rho B \widehat{\varepsilon} &= \begin{cases} \mathcal{G} & \text{I } \ B\widehat{\varepsilon} \models B' \\ \emptyset & \text{t rw s} \end{cases} \\
[\![F \wedge F']\!]_s \rho B \widehat{\varepsilon} &= [\![F]\!]_s \rho B \widehat{\varepsilon} \cap [\![F']\!]_s \rho B \widehat{\varepsilon} \\
[\![F \vee F']\!]_s \rho B \widehat{\varepsilon} &= \bigcup \{ [\![F]\!]_s \rho B_{1} \widehat{\varepsilon} \cap [\![F']\!]_s \rho B \ \widehat{\varepsilon} \mid B \widehat{\varepsilon} \models B_{1} \vee B \ \} \\
[\![\langle \tau \rangle F]\!]_s \rho B \widehat{\varepsilon} &= \left\{ \begin{array}{l} t \mid \exists \{c_i\}_I \cdot B\widehat{\varepsilon} \models \bigvee_I c_i, \forall i. \exists t \overset{b_i, \tau}{\longmapsto} t'_i \text{ w t } \ c_i \models b_i \\ \qquad \qquad \qquad \qquad \text{n } \ t'_i \in [\![F]\!]_s \rho (B \wedge c_i) \widehat{\varepsilon} \end{array} \right\} \\
[\![[\tau] F]\!]_s \rho B \widehat{\varepsilon} &= \left\{ t \mid \forall t \overset{b', \tau}{\longmapsto} t' \ \text{pr s } t' \in [\![F]\!]_s \rho (B \wedge b') \widehat{\varepsilon} \right\} \\
[\![\langle c \ x \rangle F]\!]_s \rho B \widehat{\varepsilon} &= \left\{ \begin{array}{l} t \mid \exists \{c_i\}_I . B\widehat{\varepsilon} \models \bigvee_I c_i \cdot \forall i. \exists t \overset{b_i, c \ e_i}{\longmapsto} t'_i \text{ w t } \ c_i \models b_i \\ \qquad \qquad \qquad \qquad \text{n } \ t'_i \in [\![F[e_i/x]]\!]_s \rho (B \wedge c_i) \widehat{\varepsilon} \end{array} \right\} \\
[\![[c \ x] F]\!]_s \rho B \widehat{\varepsilon} &= \left\{ t \mid \forall t \overset{b', c \ e}{\longmapsto} t' \ \text{pr s } t' \in [\![F[e/x]]\!]_s \rho (B \wedge b') \widehat{\varepsilon} \right\} \\
[\![\langle c \ \ \rangle G]\!]_s \rho B \widehat{\varepsilon} &= \left\{ t \mid \exists \{c_i\}_I . B\widehat{\varepsilon} \models \bigvee_I \right.
\end{aligned}
$$

C s F po nt ppro t ons– s ow t s $\boldsymbol{F}$ s $\boldsymbol{\mu}^{\alpha}\boldsymbol{X}.\boldsymbol{F}'$–

uppos $\boldsymbol{t} \models_{\boldsymbol{B}\hat{\varepsilon}} \boldsymbol{\mu}^{\alpha}\boldsymbol{X}.\theta\boldsymbol{F}'$– I α s t n $\boldsymbol{H}(\theta\boldsymbol{F})$ or s tr v – I α s t or n t n $\boldsymbol{H}(\boldsymbol{\mu}^{\beta}\boldsymbol{X}.\boldsymbol{F}')$ or s or $\beta < \alpha$

$$
\begin{array}{lcl}
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{B} & = & \boldsymbol{B}[\varepsilon(\boldsymbol{z})/\boldsymbol{z}] \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}_{1} \wedge \boldsymbol{F} & = & \varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}_{1} \wedge \varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}_{1} \vee \boldsymbol{F} & = & \varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}_{1} \vee \varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\langle\tau\rangle\boldsymbol{F} & = & \bigvee_{\boldsymbol{t} \overset{\boldsymbol{b}',\tau}{\longmapsto} \boldsymbol{t}'} \boldsymbol{b}' \wedge \varepsilon \triangleright \boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{F} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}[\tau]\boldsymbol{F} & = & \bigwedge_{\boldsymbol{t} \overset{\boldsymbol{b}',\tau}{\longmapsto} \boldsymbol{t}'} \boldsymbol{b}' \rightarrow \varepsilon \triangleright \boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{F} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\langle \boldsymbol{c}\ \boldsymbol{x}\rangle\boldsymbol{F} & = & \bigvee_{\boldsymbol{t} \overset{\boldsymbol{b}',\boldsymbol{c}\ \boldsymbol{e}}{\longmapsto} \boldsymbol{t}'} \boldsymbol{b}' \wedge \varepsilon \triangleright \boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{F}[\boldsymbol{e}/\boldsymbol{x}] \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}[\boldsymbol{c}\ \boldsymbol{x}]\boldsymbol{F} & = & \bigwedge_{\boldsymbol{t} \overset{\boldsymbol{b}',\boldsymbol{c}\ \boldsymbol{e}}{\longmapsto} \boldsymbol{t}'} \boldsymbol{b}' \rightarrow \varepsilon \triangleright \boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{F}[\boldsymbol{e}/\boldsymbol{x}] \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\langle \boldsymbol{c}\ \rangle\boldsymbol{G} & = & \bigvee_{\boldsymbol{t} \overset{\boldsymbol{b}',\boldsymbol{c}}{\longmapsto} (\boldsymbol{x})\boldsymbol{t}'} \boldsymbol{b}' \wedge \varepsilon \triangleright (\boldsymbol{x})\boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{G} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}[\boldsymbol{c}\ ]\boldsymbol{G} & = & \bigwedge_{\boldsymbol{t} \overset{\boldsymbol{b}',\boldsymbol{c}}{\longmapsto} (\boldsymbol{x})\boldsymbol{t}'} \boldsymbol{b}' \rightarrow \varepsilon \triangleright (\boldsymbol{x})\boldsymbol{t}'\,\mathrm{s\ t}\boldsymbol{G} \\
\varepsilon \triangleright (\boldsymbol{y})\boldsymbol{t}\,\mathrm{s\ t}\forall \boldsymbol{x}.\boldsymbol{F} & = & \forall \boldsymbol{w}.(\varepsilon \triangleright \boldsymbol{t}[\boldsymbol{w}/\boldsymbol{y}]\,\mathrm{s\ t}\boldsymbol{F}[\boldsymbol{w}/\boldsymbol{x}])\ \ \boldsymbol{w} = \boldsymbol{new}((\boldsymbol{y})\boldsymbol{t}, \varepsilon, \forall \boldsymbol{x}.\boldsymbol{F}) \\
\varepsilon \triangleright (\boldsymbol{y})\boldsymbol{t}\,\mathrm{s\ t}\exists \boldsymbol{x}.\boldsymbol{F} & = & \exists \boldsymbol{w}.(\varepsilon \triangleright \boldsymbol{t}[\boldsymbol{w}/\boldsymbol{y}]\,\mathrm{s\ t}\boldsymbol{F}[\boldsymbol{w}/\boldsymbol{x}])\ \ \boldsymbol{w} = \boldsymbol{new}((\boldsymbol{y})\boldsymbol{t}, \varepsilon, \exists \boldsymbol{x}.\boldsymbol{F}) \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{A}.(\boldsymbol{e}/\boldsymbol{z}) & = & [\varepsilon(\boldsymbol{e})/\boldsymbol{z}] \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{A} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\nu\boldsymbol{X}[\mathcal{A}]\boldsymbol{F} & = & \begin{cases} \lfloor \boldsymbol{B} \rfloor & \exists(\boldsymbol{B}\widehat{\varepsilon}', \boldsymbol{t}) \in \mathcal{A}\ \mathrm{w\ t}\ \ \boldsymbol{B}\widehat{\varepsilon}' \models \widehat{\varepsilon} \\ \nu\boldsymbol{X}_{\boldsymbol{t}\varepsilon}.(\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}[\nu\boldsymbol{X}[\mathcal{A}^{+}]\boldsymbol{F}/\boldsymbol{X}]) & \mathrm{ot\ \ rw\ s} \end{cases} \\
\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\mu\boldsymbol{X}[\mathcal{A}]\boldsymbol{F} & = & \begin{cases} \sim & \exists(\boldsymbol{B}\widehat{\varepsilon}', \boldsymbol{t}) \in \mathcal{A}\ \mathrm{w\ t}\ \ \boldsymbol{B}\widehat{\varepsilon}' \models \widehat{\varepsilon} \\ (\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\boldsymbol{F}[\mu\boldsymbol{X}[\mathcal{A}^{+\mu}]\boldsymbol{F}/\boldsymbol{X}]) & \mathrm{ot\ \ rw\ s} \end{cases}
\end{array}
$$

w r $\mathcal{A}^{+} = \mathcal{A} \cup ((\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\nu\boldsymbol{X}[\mathcal{A}]\boldsymbol{F})\widehat{\varepsilon}, \boldsymbol{t})$ n $\mathcal{A}^{+\mu} = \mathcal{A} \cup ((\varepsilon \triangleright \boldsymbol{t}\,\mathrm{s\ t}\mu\boldsymbol{X}[\mathcal{A}]\boldsymbol{F})\widehat{\varepsilon}, \boldsymbol{t})$–

Figure 5.9. t onstru t on or s or s nt s

$$
\begin{aligned}
\boldsymbol{DApps}(\boldsymbol{B}) &= \boldsymbol{DApps}(\boldsymbol{X}) \quad = \emptyset \\
\boldsymbol{DApps}(\boldsymbol{F}_1 \wedge \boldsymbol{F}\ ) &= \boldsymbol{DApps}(\boldsymbol{F}_1 \vee
\end{aligned}
$$

n us n μ un o n s n [τ] ru s n [i] ru – ow ot – n – n r u to t s n u nt $\hat{\varepsilon} \vdash t$ F [

soun n ss o t proo t n qu FI o s pr n p u to t o n u t v n t on o s r t –Cons r t two nts

$$X \Longleftarrow \alpha.X \quad n \quad Y \Longleftarrow \alpha.Y + \alpha.X.$$

wou pp onv n ours v s t t X n Y r s r r son n s r to t r son n w wou us or FI– wou ons r α ov r Y n t t w t ov r X– For p $Y \xrightarrow{\alpha} Y$ s t $X \xrightarrow{\alpha} X$ n r ut not or w ssu t t X n Y r s r–For w wou onstru t $\mathcal{R} = \{(X,Y),(X,X)\}$ s w tn ss n s u t on n t n pp to t o n u t on pr n p w t s us t t $\mathcal{R} \subseteq$ –

FI ru us n u r t on $\{X_i \Longleftarrow p_i\}_I$ s s r n sp r t to t pr v ous ru – ***simultaneously*** st s t pot s s t t

$$\vdash q_i = p_i[q/X]$$

or t s $\{q_i\}_I$ n u r r t ons $\{p_i\}_I$–Fro t s w n r t t $q_1 = X_1$–

purpos o t s pt r s to nv st t t us o un qu po nt n u t on n v u p ss n n u n or r to r t r s s u t on qu v n s ov r ss o r urs v n nts– p rt u r n u w ons r s v u p ss n CC ut w nt p t t t t s uss on w pp to w r ss o pro ss n u s n u n CB –

w r $f_i \equiv \lambda x_i.u_i$ n $\{X_i \Longleftarrow \lambda x_i.t_i\}$ s u r r t on– s ru n v st t s ov s unsoun –H nn ss n n [4] prov t o ow n p to s ow t s– t

$$Y \Longleftarrow \lambda x.c\ |x|.c\ z.Y(z)$$

w r $\boldsymbol{D}$ s r t on– r t on v n on s qu nt s us to nt ow to un o r urs v n onst nt– t s t un o n ru

$$\frac{}{\vdash_{\boldsymbol{D}} \quad \rhd \boldsymbol{X} = \boldsymbol{f}} \qquad \boldsymbol{X} \Longleftarrow \boldsymbol{f} \in \boldsymbol{D}$$

us s $\boldsymbol{D}$ to t n $\boldsymbol{f}$ t o o $\boldsymbol{X}$– s ou v w t out v n to rr t n $\boldsymbol{D}$ roun s p ssu n n un r n r t on $\boldsymbol{D}$ n r rr n to t s n s on t ons– How v r t r t on s n p t on s qu nt n or r to ow r t ons to t n proo –For p suppos proo w r tt pt n s to t su o

$$\vdash_{\boldsymbol{D}} \boldsymbol{b} \rhd \boldsymbol{t} = \boldsymbol{u}$$

w r $\boldsymbol{t}$ n $\boldsymbol{u}$ r ot n $\mathcal{T}_{\boldsymbol{D}}$– on v t v to ntro u n w nt onst nt $\boldsymbol{X}$ t n n $\boldsymbol{D}$

E I $\quad \dfrac{}{\vdash_D \quad \triangleright t = t} \qquad \dfrac{\vdash_D b \triangleright t = u}{\vdash_D b \triangleright u = t} \qquad \dfrac{\vdash_D b \triangleright t = u \quad \vdash_D b \triangleright u = v}{\vdash_D b \triangleright t = v}$

AXI $\quad \dfrac{t = u \in \text{A} \quad \text{o s}}{\vdash_D \quad \triangleright t = u}$

C G $\quad \dfrac{\vdash_D b \triangleright t_1 = u_1 \quad \vdash_D b \triangleright t \quad = u}{\vdash_D b \triangleright t_1 + t \quad = u_1 + u}$

α C $\quad \dfrac{}{\vdash_D \quad \triangleright c \;\; x.t = c \;\; y.t[y/x]} \qquad y \notin fv(t)$

I $\quad \dfrac{\vdash_D b \triangleright t = u}{\vdash_D b \triangleright c \;\; x.t = c \;\; x.u} \qquad x \notin fv(b)$

$\dfrac{b \models e = e' \quad \vdash_D b \triangleright t = u}{\vdash_D b \triangleright c \;\; e.t = c \;\; e'.u}$

A $\quad \dfrac{\vdash_D b \triangleright t = u}{\vdash_D b \triangleright \tau.t = \tau.u}$

G A D $\quad \dfrac{\vdash_D b \wedge b' \triangleright t = u \quad \vdash_D b \wedge \neg b' \triangleright \mathbf{n} \quad = u}{\vdash_D b \triangleright b' \rightarrow t = u}$

C $\quad \dfrac{\vdash_D b' \triangleright t = u}{\vdash_D b \triangleright t = u} \qquad b \models b'$

CA E $\quad \dfrac{\vdash_D b_1 \triangleright t = u \ldots \vdash_D b_n \triangleright t = u}{\vdash_D \bigvee}$

I $\qquad \dfrac{\vdash_{D} b \rhd t = u}{\vdash_{D \cup E} b \rhd t = u}$

E $\qquad \dfrac{\vdash_{D \cup E} b \rhd t = u}{\vdash_{D} b \rhd t = u} \qquad t, u \in \mathcal{T}_{D}$

FIX $\qquad \dfrac{}{\vdash_{D} \quad \rhd X = f} \qquad X \Longleftarrow f \in D$

FI $\qquad \dfrac{\forall i \in I \quad \vdash_{D} \quad \rhd g_i = f_i[g/X]}{\vdash_{D \cup E} \quad \rhd g_1 = X_1} \qquad$ w r $E = \{X_i \Longleftarrow f_i\}_I$ s u r r t on

λ I $\qquad \dfrac{\vdash_{D} b \rhd f(x) = g(x)}{\vdash_{D} b \rhd f = g} \qquad x \notin fv(b)$ n $x_i \neq x_j$ or $i \neq j$

λ E $\qquad \dfrac{\vdash_{D} b \rhd f = g}{\vdash_{D} b \rhd f(e) = g(e')} \qquad b \models e = e'$

β $\qquad \dfrac{}{\vdash_{D} \quad \rhd (\lambda x.t)(e) = t[e/x]} \qquad x$

Figure 6.2.

oo [4]

onv rs o t s s t nt r st n propos t on o o pr ■

Let $D_1 = \{X_i \Longleftarrow f_i\}_I$ and $D = \{Y_j \Longleftarrow g_j\}_J$ be standard declarations such that $X_1(e_1)$ $\overset{b}{L}$ $Y_1(e'_1)$. Then there exists a standard declaration $E = \{Z_{ij} \Longleftarrow h_{ij}\}_{I\times J}$ such that

$$\mathcal{A} \vdash_{D_1 \cup E} b \rhd X_1(e_1) = Z_{11}(e_1, e'_1)$$

and

$$\mathcal{A} \vdash_{D \cup E} b \rhd Y_1(e'_1$$

Furt r or t s p rs o t r s s t s t pot s s o –– so w pp t s r
to o t n t p rt t on B_{ijkl}– ow or $b' \in B_{ijkl}$ w n

$$I^{b'} = \left\{ (p,q) \mid b' \models \alpha_{ikp} = \beta_{jlq} \text{ n } X_{f(ikp)}(e_{ikp}) \; {}^{b'}_{L} \; Y_{g(jlq)}(e_{jlq}) \right\}.$$

prop rt s o B_{ijkl} v n –– nsur t t $I^{b'}$ s tot n sur t v r t on on $P_{ik} \times Q_{jl}$– us t s so n ppl t on o A or I w v us

$$\vdash_{D \cup E} b' \rhd \beta_{jlq}.Y_{g(jlq)}(e_{jlq}) = \alpha_{ikp}.Y_{g(jlq)}(e_{jlq}).$$

r p t t s or *q* n to o t n – –
s qu nt – s s t s r to st s us n s r r u nt ow n to t t t t
E s **built in favour** o D_1 n $I^{b'}$ s tot – ■

o ***(Completeness) Let t and u be regular terms with identifiers in D, where D is a regular, guarded, declaration. Then***

$$t \overset{b}{\underset{L}{}} u \text{ implies } \mathcal{A} \vdash_D b \rhd t = u.$$

oo rst tr ns o *t* n *u* nto r t ons us n ropos t on – – – s s r t ons $D_1 = \{X_i\}$ n $D = \{Y_j\}$ su t t

$$\vdash_{D \cup D_1} \rhd t = X_1(x) \quad n \quad \vdash_{D \cup D} \rhd u = Y_1(y)$$

w r $fv(t) = x$ n $fv(u) = y$– or ov r w ssu t t D_1 n D r st n r o s n v s o nt p r t rs– now t t X_1

- w n v r $p \xrightarrow{\alpha} p'$ $\alpha \neq c$ t $q \overset{\alpha}{\Longrightarrow} q'$ or so q' su t t $(p', q') \in \mathcal{R}$

w t s tr on t ons or q– t $p \approx_L q$ t r sts t w s u t on $\mathcal{R}$ su t t $(p,q) \in \mathcal{R}$– w rop t su p L unt w s uss t orr spon n **early** qu v n – ***Lat obs rvat on con ru nc*** o p ss n CC = s t r t on n $p = q$

- w n v r $p \xrightarrow{c} (x)t$ t n $q \Longrightarrow (y)u$ or so $(y)u$ su t t or $v \in Val$ t r s q' su t t $u[v/y] \overset{\varepsilon}{\Longrightarrow} q'$ n $t[v \ldots] \approx q'$–
- w n v r $p \xrightarrow{\alpha} p'$ $\alpha \neq c$ t $q \overset{\alpha}{\Longrightarrow} q'$ or so q' su t t $p' \approx q'$

on w t t s on t ons on q

E t ns v us o s o s or v u p ss n CC w us or t r n r o t s pt r– us w n t w o s u t ons n t s o on ru n or t s n u –

s o v rs on o t w tr ns t on r t on $\Longrightarrow$ s n s o ows

- $t \overset{,\varepsilon}{\Longrightarrow} t$
- $t \overset{b,\alpha}{\longmapsto} u$ pr s $t \overset{b,\alpha}{\Longrightarrow} u$
- $t \overset{b,\tau}{\longmapsto}\overset{b',\alpha}{\Longrightarrow} u$ pr s $t \overset{b \wedge b',\alpha}{\Longrightarrow} u$
- $t \overset{b,\tau}{\Longrightarrow}\overset{b',\tau}{\longmapsto} u$ pr s $t \overset{b \wedge b',\tau}{\Longrightarrow} u$
- $t \overset{b,c\,e}{\Longrightarrow}\overset{b',\tau}{\longmapsto} u$ pr s $t \overset{b \wedge b',c\,e}{\Longrightarrow} u$

uppos $= \{S^b\}$ s oo n r o r t ons–D n $\mathcal{WSB}(\ )$ to t o r t ons su t t

$(t,u) \in \mathcal{WSB}(\ )^b$ w n v r $\ldots t'$ t r sts v r z su t t $z \notin fv(b,t,u)$ n $b \wedge b_1$ p rt t on B su or $b \ldots z \notin fv(b')$ n t r sts $u \overset{b\ ,\beta}{\Longrightarrow} u'$ su t t $b' \models b$ n

- α s τ t n $\beta \equiv \tau$ n $(t',u') \ldots S^{b'}$
- α s $c\,e$ t n $\beta \equiv c\,e'$ w t $\ldots e = e'$ n $(t',u') \in S^{b'}$
- α s $c\,x$ t n $\beta \equiv c\,y$ or so $\ldots$ n t r sts b' p rt t on B' su t t or $b'' \in B'$ t r s u'' su t t $u'[z/y] \ldots u''$ w t $b'' \models b'$ n $(t'[z/x], u'') \in S^{b''}$–

$\{S^b\}$ ***at w a sy b s u at on*** $S^b \subseteq \mathcal{WSB}(\ )^b$ or b n not t r st su $\{\approx^b\}$– n w now us t n t on o $\approx^b$ to n $=^b$ t r st on ru n ont n n $\approx^b$

$t =^b u$ w n v r $t \overset{b_1,\alpha}{\longmapsto} t'$ t r sts v r z su t t $z \notin fv(b,t,u)$ n $b \wedge b_1$ p rt t on B su t t or $b' \in B$ $z \notin f\ldots$ n t r sts $u \overset{b\ ,\beta}{\Longrightarrow} u'$

p Suppose we have standard, saturated declarations

$$X_i \Longleftarrow \lambda x_i. \sum_{k \in K_i} c_{ik} \rightarrow \sum_{p \in P_{ik}} \alpha_{ikp}.X_{f(ikp)}(e_{ikp})$$

and

$$Y_j \Longleftarrow \lambda y_j. \sum_{l \in L_j} d_{jl} \rightarrow \sum_{q \in Q_{jl}} \beta_{jlq}.X_{g(jlq)}(e_{jlq}).$$

Also suppose that $X_i(x_i) \approx^{b \wedge c_{ik} \wedge d_{jl}} Y_j(y_j)$, *then* $t_{ik} \approx^{b \wedge c_{ik} \wedge d_{jl}} u_{jl}$ *where*

$$t_{ik} \equiv \sum_{P_{ik}} \alpha_{ikp}.X_{f(ikp)}(e_{ikp})$$

and

$$u_{jl} \equiv \sum_{Q_{jl}} \beta_{jlq}.Y_{g(jlq)}(e_{jlq}).$$

Moreover there exist disjoint $b \wedge c_{ik} \wedge d_{jl}$*-partitions* B^c_{ijkl}, B^c_{ijkl} *and* B^τ_{ijkl} *such that*

- *For each* $b' \in B^c_{ijkl}$ *and for each* $p \in P_{ik}$ *such that* $\alpha_{ikp} \equiv c\ e$, *there exists a* $q \in Q_{jl}$ *such that* $\beta_{jlq} \equiv c\ e'$ *with* $b' \models e = e'$ *and* $X_{f(ikp)}(e_{ikp}) \approx^{b'} Y_{g(jlq)}(e_{jlq})$.
- *For each* $b' \in B^\tau_{ijkl}$ *and for each* $p \in P_{ik}$ *such that* $\alpha_{ikp} \equiv \tau$, *then either* $X_{f(ikp)}(e_{ikp}) \approx^{b'} Y_j(y_j)$ *or there exists a* $q \in Q_{jl}$ *such that* $\beta_{jlq} \equiv \tau$ *with* $X_{f(ikp)}(e_{ikp}) \approx^{b'} Y_{g(jlq)}(e_{jlq})$
- *For each* $b' \in B^c_{ikjl}$ *and for each* $p \in P_{ik}$ *such that* $\alpha_{ikp} \equiv c\ w$, *there exists a* $q \in Q_{jl}$ *such that* $\beta_{jlq} \equiv c\ w$ *and there exists a disjoint* b'*-partition,* $B'_{p,b'}$ *such that for each* $b'' \in B'_{p,b'}$ *we have* $X_{f(ikp)}(e_{ikp}) \approx^{b''} Y_{g(jlq)}(e_{jlq})$ *or* $Y_{g(jlq)}(e_{jlq}) \overset{d,\tau}{\longmapsto} Y_{j(b'')}(e(b''))$ *for some* $j(b'')$ *and* $e(b'')$ *with* $b'' \models d$ *and* $X_{f(ikp)}(e_{ikp}) \approx^{b''} Y_{j(b'')}(e(b''))$

(Similar conditions for each $q \in Q_{jl}$ *follow by symmetry).*

oo. w wr t b_{ijkl} s n r v t on or $b \wedge c_{ik} \wedge d_{jl}$. now t t $X_i(x_i) \approx^{b_{ijkl}} Y_j(y_j)$ so s ontn ss o t c_{ik}s n d_{jl}s w s s t t $t_{ik} \approx^{b_{ijkl}} u_{jl}$. C oos so $p \in P_{ik}$ n ons r t t r s s o t o o α_{ikp}.

C α_{ikp} s $c\ e$. now t t $X_i(x_i) \approx^{b_{ijkl}} Y_j(y_j)$ n t t $X_i(x_i) \overset{c_{ik}, c\ e}{\longmapsto}$

r r $\{q_1, \ldots, q_m\}$ s t s t o r $q \in Q_{jl}$ su t t β_{jlq} s o t o c e t n w r t

$$E^c = \left\{ \bigwedge_{1 \le i \le m} b_i \mid b_i \in B^c_{q_i}, 1 \le i \le m \right\}.$$

p rt t on B^c_{ijkl} w r ont n on un t ons o oo ns os n p rw s ro D^c n E^c – For r t s s

$$B^c_{ijkl} = \left\{ b \wedge b' \mid b \in D^c, b' \in E^c \right\}.$$

It s s p tt r to t t B^c_{ijkl} s n b_{ijkl} p rt t on– or ov r v n n $b' \in B^c_{ijkl}$ n n $p \in P_{ik}$ su t t $\alpha_{ikp} \equiv c$ e w now t t p s r pr s nt n b' on o t p_is– s ns t t $b' \models b_p$ or so $b_p \in B^c_p$ n t r qu r prop rt s o b' o ow s – n t p rt t ons B^{τ}_{ijkl} n B^c_{ijkl} n n nt nn r–For $b' \in B^c_{ijkl}$ n $p \in P_{ik}$ w now t t $b' \models b_p$ or so $b_p \in B^c_p$ n t t b_p s urt r p rt t on nto B'_{p,b_p}– tu n b' p rt t on $B'_{p,b'}$ r ut t s s o t n s pr n n $\left\{ b'' \wedge b' \mid b'' \in B'_{p,b_{p_S}} \right.$

rt o ows s ow n

$$\vdash b_i \rhd \sum_{i\in I} b_i \rightarrow \tau.u_i = \tau.\sum_{i\in I} b_i \rightarrow u_i$$

or $i \in I$ t r sur o ows CA E– s s s pr to st r s not n

$$\begin{aligned}
\vdash b_i \rhd \textstyle\sum_{j\in I} b_j \rightarrow \tau.u_j &= \textstyle\sum_{j\in I} b_i \wedge b_j \rightarrow \tau.u_j \\
&= b_i \rightarrow \tau.u_i \\
&= \tau.b_i \rightarrow u_i \\
&= \tau.\textstyle\sum_{j\in I} b_i \wedge b_j \rightarrow u_j \\
&= \tau.\textstyle\sum_{j\in I} b_j \rightarrow u_j.
\end{aligned}$$

■

o ***Let*** $D_1 = \{X_i \Leftarrow g_i\}_I$ ***and*** $D = \left\{Y_j \Leftarrow g'_j\right\}_J$ ***be standard, saturated, strongly guarded declarations such that*** X_1 ***does not appear in any*** g_i ***and*** Y_1 ***does not appear in any*** g'_j***. If*** $X_1(e_1) =^b Y_1(e'_1)$ ***then there exists a standard declaration*** $E = \{Z_{ij} \Leftarrow h_{ij}\}_{I\times J}$

$$I_{b'}^{c} = \Big\{$$

r st st p or s us $p \in P_{ik}$ su t t α_{ikp} s so c e pp rs n $I^{c}_{b'}$ –

r r w now oos n r tr r $b' \in B^{c}$

n us T to o t n or b'

$$\vdash b' \rhd c\ \ w.X_{f(ikp)}(e_{ikp}) = c\ \ w.X_{f(ikp)}(e_{ikp}) + c\ \ w.\sum_{b''\in B'_{q,b'}} b'' \to X_{i(b'')}(e(b''))$$

w n n t r st o t^c n us n 1 v s us

$$\vdash b' \rhd t^c\ = t^c\ + V\ [f/Z].$$

r or w v our r su t us

$$\begin{aligned} \vdash b' \rhd V^c_{ijkl}[f/Z] &= V_1[f/Z] + V\ [f/Z] \\ &= t^c\ + V\ [f/Z] \\ &= t^c \end{aligned}$$

F n w s ow 1 onstr t n

$$\vdash b' \rhd V^{\tau}_{ijkl}[f/Z] = t^{\tau} + \sum_{\substack{k\in K_i \\ l\in L_j}} \sum_{b'\in B^{\tau}_{ijkl}} \sum_{(\tau,q)\in I^{\tau}_{b'}} b' \to \tau.X_i$$

- α s τ t n β ≡ τ n $t' \approx^{b'} u'$
- α s c e t n β ≡ c e' w t $b' \models e = e'$ n $t' \approx^{b'} u'$
- α s c x t n β ≡ c y or so y n $t'[z/x] \approx^{b'} u'[z/y]$–

r r o ours s tr on t ons on u r qu r n t ov n t ons–
nsurpr s n t on n s to t proo s st w r

v t o s or r str t on

$$
\begin{aligned}
- \backslash \boldsymbol{c} &= - \\
(\boldsymbol{X}+\boldsymbol{Y})\backslash \boldsymbol{c} &= \boldsymbol{X}\backslash \boldsymbol{c}+\boldsymbol{Y}\backslash \boldsymbol{c} \\
(\boldsymbol{b}\rightarrow \alpha.\boldsymbol{X})\backslash \boldsymbol{c} &= \begin{cases} - & \alpha \text{ s } \boldsymbol{c}\ \boldsymbol{e} \text{ or } \boldsymbol{c}\ \boldsymbol{x} \\ \boldsymbol{b} & \end{cases}
\end{aligned}
$$

G v n r t on $\boldsymbol{D} = \left\{ \boldsymbol{X_i} \Longleftarrow \lambda \boldsymbol{x_i}. \sum_{\boldsymbol{k} \in \boldsymbol{K_i}} \alpha_{\boldsymbol{ik}}.\boldsymbol{X}_{f(\boldsymbol{ik})}(\boldsymbol{e_{ik}}) \right\}_{\boldsymbol{I}}$ t n w n n t ***regular*** r t on $\boldsymbol{D}\backslash\boldsymbol{c}$ s

$$\left\{ \boldsymbol{Z_i} \Longleftarrow \lambda \boldsymbol{x_i}. \sum_{\alpha_{ik} \neq \boldsymbol{c} \ ,\boldsymbol{c}} \alpha_{\boldsymbol{ik}}.\boldsymbol{Z}_{f(\boldsymbol{ik})}(\boldsymbol{e_{ik}}) \right\}_{\boldsymbol{I}} .$$

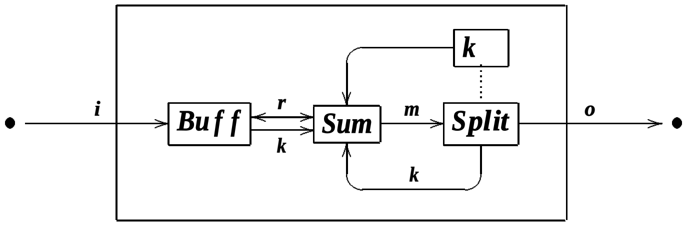


Figure 6.4. p nt t on o ***Spec***

$p \equiv X_i(e)$ n $q \equiv C_i'[e/x_i]$– uppos t t $p \xrightarrow{\alpha} p'$ or so p' so t t $[[b_{ik}[e/x_i]]] =$ or so $k \in K_i$ w t $\alpha = \alpha_{ik}[e/x_i]$ n $p' \equiv X_{f(ik)}(e_{ik}[e/x_i])$– now t t

$$q \xrightarrow{\tau} C_i''[e/x_i] \xrightarrow{\tau} C_i[e/x_i] \xrightarrow{\alpha} q'$$

w r $q' \equiv C'_{f(ik)}[e$

n ss r –

s ns s ts o on r t v u s n s n s str t v u s– r n n ppro s n t nt rpr t t on o t un t ons n t t s n tur – ur ppro ows or n o ***precise*** nt rpr t t on on nt rpr t t on o un t ons s t n t str t on on v u s– o t str t n n f_A o un t on f o r t on s n s

$$f_A(V) = \{ f(v) \mid v \in V \}$$

w r V n n str t v u s s t o on r t v u s ro ***Val***– us w r un to r p so o t n ts o n r str t on–For pr w w s to onstr t ro r o o t pro ss $p(x)$ w r

$$p \Longleftarrow \lambda y.c\ y.p(y+1)$$

t n t s o s nt s n u s ***abstract values*** ons r n t on r t v u s t t x t –In t x ou n v u w w r pr s nt t s t ***Val***– s on output ro $p(x$

nt r.s o o nst nt t t r nt v u t r un o n – For pr t pro ss

$$X \Longleftarrow \lambda x.(X(x+1)+a\ x.\ ),$$

w n nst nt t t s n n nt r n n s or r p – Gu r r urs ons w s v nt r n n s or r p s so t s r t t X nnot r u to u r r t on– wo t ppro s spr n to

B o p

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